

# Modeling the Distribution of Tax Evasion: Implications for the Tax Gap

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## Abstract

Understanding the drivers of tax evasion is essential for designing policies that address the persistent gap between taxes owed and collected. This paper develops a dynamic heterogeneous-agent model in which taxpayers choose between simple and sophisticated evasion under enforcement risk. Sophisticated evasion is accessible only for high-income individuals who can afford its fixed costs and exploit legal ambiguities in the tax system, generating a productivity-dependent threshold that shapes non-compliance across the income distribution. Calibrated to U.S. data, the model replicates both the aggregate tax gap and its concentration at the top. Simulations show that traditional deterrence tools, such as audits and fines, substantially reduce simple evasion but have limited influence on sophisticated evasion, whose incentives are largely insulated from enforcement intensity. Consequently, the remaining tax gap is increasingly composed of harder-to-detect behavior among high-income taxpayers. Greater income dispersion expands the set of individuals who can use sophisticated avoidance, flattening the Laffer curve and reducing the government's maximum attainable revenue under imperfect detection. The results highlight that effective policy must target the mechanisms that allow sophisticated evasion to appear legitimate, such as legal ambiguity and weak fine-enforcement credibility, rather than relying solely on standard deterrence instruments.

**Keywords:** tax evasion; avoidance; fiscal policy; heterogeneity; tax gap; random audits; enforcement

**JEL Codes:** H26, H21, H30, E62, D31

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# 1 Introduction

Improving tax compliance, particularly among high-income individuals, remains a formidable challenge. The tax gap, the share of legally owed taxes that goes uncollected, has remained persistently high, at roughly 15% in the U.S. and 12% in Europe over the past decade (IRS, 2019; Murphy, 2021). Despite incremental progress from recent enforcement initiatives, the literature shows that high-income individuals continue to minimize their tax liabilities<sup>1</sup> and still hold substantial portions of their assets in tax havens, where offshore positions have remained near 10% of global GDP for more than a decade. They also respond more strongly to tax changes than the average taxpayer (Rubolino and Waldenström, 2019), and face substantially lower effective income tax rates than the rest of the population,<sup>2</sup> underscoring the distinct incentives and opportunities available at the top of the income distribution. Against this backdrop, this paper raises two fundamental questions: What drives the differences in tax-evasion behavior across the income distribution? And how do these differences shape the government’s ability to collect revenues over time?

We start from the notion that these disparities persist because high-income individuals can shift from easily-detected under-reporting to costly, sophisticated evasion strategies that exploit legal gray areas to obscure income. These strategies blur the line between legal avoidance and illegal evasion, undermining the effectiveness of traditional enforcement tools like audits and fines.

To capture these dynamics, we embed simple and sophisticated evasion directly into a dynamic heterogeneous-agent environment. Agents face income risk, accumulate capital, and choose evasion levels knowing that the government cannot always distinguish illegal evasion from legally defensible avoidance. Unlike static representative-agent models, this framework allows enforcement risk and income heterogeneity to jointly determine which evasion margin

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<sup>1</sup>Total evasion as a percentage of true income has been documented to increase with income across Scandinavia, Colombia, the Netherlands, and the United States (Alstadsæter et al., 2019; Londoño-Vélez and Ávila Mahecha, 2021; Leenders et al., 2023; Johns and Slemrod, 2010; Guyton et al., 2023).

<sup>2</sup>For example, billionaires’ effective income tax rates are as low as 8% in the U.S. (The White House, 2021) and 2% in France (EU Tax Observatory, 2023).

is operative at each income level. This structure helps explain why the tax gap remains concentrated at the top and why aggregate non-compliance has proven resistant to repeated policy efforts

To study these mechanisms, we develop a dynamic model in which risk-averse agents facing income shocks choose between *simple* and *sophisticated* forms of tax evasion. Sophisticated evasion involves a fixed cost and succeeds only when the tax authority fails to distinguish it from legal avoidance; enabling its users to exploit legal ambiguities in the tax code. The government audits taxpayers randomly and imposes fines when evasion is detected—conditional on having the capacity to credibly enforce them.

The model features heterogeneity in income and capital accumulation, allowing it to trace how evasion decisions vary with income and over time. These evasion decisions, in turn, shape the size and structure of the tax gap. We calibrate the model to U.S. data, targeting key empirical moments such as the level and composition of the tax gap documented by Guyton et al. (2023). We then simulate a range of enforcement policies, including audit frequency, fine size, and fine enforcement capacity, to evaluate how these tools interact with evasion strategies and income dynamics.

The model shows that the government’s ability to reduce the tax gap, or its revenue collecting capacity, is structurally constrained by the prevalence of costly, sophisticated evasion and by the distribution of income and wealth across agents.

While individual evasion rises monotonically with income in the model, its contribution to the tax gap does not. Instead, the composition of evasion shifts at the top toward sophisticated strategies that are difficult to detect or penalize under imperfect enforcement. As this margin expands, a growing share of potential revenue falls into a region where traditional deterrence tools, audits and fines, have little traction. Simple evasion remains responsive to enforcement across the distribution, but its aggregate importance is limited precisely because the tax gap is increasingly shaped by sophisticated behavior concentrated among a small group of high-income taxpayers.

Greater income dispersion amplifies this constraint. As more taxpayers surpass the fixed-cost threshold for sophisticated evasion, a larger portion of the tax base transitions into the hard-to-enforce region. This composition effect flattens the revenue-collection Laffer curve and lowers the peak of the revenue-maximizing tax rate under imperfect detection.

These results are driven by two key mechanisms: a *feedback loop* between evasion and income, and a *threshold effect* from the fixed cost of sophisticated evasion.

First, the *feedback loop* emerges because wealthier agents are less sensitive to enforcement due to decreasing relative risk aversion inducing a greater ability to absorb enforcement risks, modeled by a Hyperbolic Absolute Risk Aversion (HARA) utility preference. These agents accumulate capital more rapidly, partly by evading taxes, and use that capital to fund future evasion, which accelerates their income growth and further undermines deterrence. The result is a reinforcing cycle: higher income leads to more evasion, which leads to higher income.

Second, a *threshold effect* arises from the fixed cost of sophisticated evasion and the tax authority's limited ability to distinguish avoidance from evasion. Only high-income taxpayers can cover the fixed cost, and when they do, the legal ambiguity of these schemes sharply limits the credible enforceability of fines even conditional on audit. As income dispersion rises, more taxpayers cross this threshold, pushing a growing share of evasion into the legally ambiguous region where deterrence tools have little impact.

Together, these mechanisms explain why traditional enforcement tools lose effectiveness at the top of the distribution and why the legal ambiguity surrounding sophisticated evasion imposes a structural limit on the state's ability to close the tax gap. Addressing this dynamic requires more than marginal changes to audits or fines—it requires targeted policies that reduce legal ambiguity and strengthen fine enforcement capacity.

## 1.1 Related Literature

Empirical studies consistently show that tax evasion increases with income, with wealthier individuals disproportionately contributing to the tax gap and highlight the need for better data reporting requirements on rich individuals to adequately estimate the aggregate impact of tax enforcement policies. Despite the implementation of global tax enforcement initiatives such as the Common Reporting Standard (CRS) and the Foreign Account Tax Compliance Act (FATCA), their impact on enhancing tax compliance has often been limited, short-lived, or unclear (De Simone et al., 2020). Offshore wealth still accounts for approximately 10% of global GDP (Guyton et al., 2020), and tax gaps in many countries, such as the United States, remain persistently high (IRS, 2019) as its rich citizens continue to circumvent new tax regulations by engaging in more sophisticated evasion strategies. For example, to circumvent FATCA’s enhanced reporting requirements on income and assets held abroad, rich U.S. individuals engage in new evasion strategies such as ‘round-tripping’, where their assets are transferred and hidden to foreign accounts and then re-invested back in US Securities (Hanlon et al., 2015), or by simply investing in other non-financial assets, such as real estate, to circumvent FATCA’s new reporting requirements (De Simone et al., 2020). These persistent gaps underscore a critical weakness in existing enforcement policies: their inability to address the dynamic and heterogeneous nature of evasion practices over time. This highlights the need for a theoretical framework that captures the interplay between income distribution, enforcement mechanisms, and the evolution of evasion strategies to better inform policy design. The framework proposed in this paper addresses these dynamics.

Building on these empirical findings, this paper draws on a rich body of work in tax compliance to provide such a framework. Foundational models by Allingham and Sandmo (1972); Yitzhaki (1987); Mayshar (1991); Slemrod and Yitzhaki (2002) established tax evasion as a static trade-off between the potential gains from evasion and the risks of detection and penalties. While these models provided critical insights into compliance behavior, they largely assumed homogeneity among taxpayers and overlooked the evolving nature of evasion

decisions. Subsequent work has emphasized the need to analyze how different enforcement tools, such as audits, fines, and third-party reporting, affect taxpayers unevenly across income groups (Kleven et al., 2011; Levaggi and Menoncin, 2016; Keen and Slemrod, 2017; Boning et al., 2023). Complementing this, Alstadsæter et al. (2019); Di Nola et al. (2021); Guyton et al. (2023) highlighted the growing role of offshore wealth and sophisticated evasion strategies among high-income taxpayers, underscoring the challenges these practices pose to enforcement mechanisms. However, much of this research has focused on either static frameworks or specific empirical patterns, leaving key questions about the dynamic evolution of evasion strategies unanswered. The dynamic model presented in this paper addresses these questions.

Recent advancements have sought to address these gaps, particularly in the context of high-income taxpayers. Gamannossi degl’Innocenti et al. (2022) introduced a dynamic model in which wealthier households shift from simple evasion to legal avoidance, showing that total evasion increases with income. While their work links evasion to aggregate tax revenues, it does not explicitly examine the distributional effects of enforcement policies. Similarly, Guyton et al. (2023) demonstrated that sophisticated evasion substitutes for simple evasion as taxpayers surpass a wealth threshold, with theoretical evidence suggesting that higher audit probabilities do not deter sophisticated strategies. The framework presented in this paper builds on these findings by incorporating dynamic enforcement mechanisms and modeling the transition to sophisticated evasion as a function of capital accumulation over time. By explicitly linking income heterogeneity, time dynamics, and policy effectiveness, this paper provides a comprehensive understanding of how tax evasion evolves and its implications for enforcement strategies.

These studies underscore the importance of modeling shifts in evasion behavior across the income distribution, but they stop short of fully capturing the dynamic feedback between evasion strategies, enforcement capacity, and capital accumulation. This paper extends the existing literature by developing a unified framework that endogenizes simple and sophis-

ticated evasion choices as a function of income dynamics and enforcement credibility. By explicitly linking evasion choices to heterogeneous income paths and the government’s limited ability to distinguish avoidance from evasion, we provide new insight into the persistence of the tax gap and the limits of traditional enforcement tools.

In **Section 2**, we develop a continuous-time heterogeneous-agent model in which taxpayers optimally choose between simple and sophisticated forms of evasion in response to income shocks and enforcement risk. **Section 3** describes the numerical methodology and calibration strategy, using U.S. data to match key empirical moments from the tax gap literature. **Section 4** presents the main results, highlighting how the model replicates observed patterns of evasion and explores the short- and long-run effects of various enforcement policies. **Section 5** discusses the mechanisms driving the results, policy implications, and the role of enforcement capacity. Finally, **Section 6** concludes with a summary of findings and avenues for future research.

## 2 Model

This section introduces a continuous-time heterogeneous-agent model in which agents make dynamic decisions about consumption, savings, and tax evasion. Agents face idiosyncratic income risk and can choose between two distinct forms of evasion: simple, which is easily punishable when detected, and sophisticated, which is costly and exploits legal ambiguity to avoid fines. The model features endogenous capital accumulation, risk-averse preferences with decreasing relative risk aversion, and a government that levies taxes, conducts random audits, and imposes fines based on its enforcement capacity. These features allow us to analyze how evasion behavior evolves over time and across the income distribution, and how it contributes to the tax gap.

## 2.1 Setting

Time  $t \in [0, \infty)$  is continuous. A unit mass of entrepreneurial households, born with heterogeneous productivity  $a_i$  drawn from distribution  $f_a(a; \sigma_a)$ , populate the economy. They have finite lifespans and their deaths are modeled as the first jump time of a Poisson process with intensity  $\phi$ . Accordingly, each household has an expected lifespan at birth of  $1/\phi$ . When one household dies, it loses its whole net worth, and a new household enters the economy to preserve the unit mass.

The optimal choices of a continuum of agents who enjoy utility from the inter-temporal consumption of a single private good  $c_t$  are modeled with the following Hyperbolic Absolute Risk Aversion (HARA) preferences:

$$U(c_t) =: \mathbb{E}_{t0} \left[ \int_{t0}^{\infty} e^{-\rho(t-t0)} \frac{(c_t - c_m)^{1-\gamma}}{1-\gamma} dt \right] \quad (1)$$

in which  $c_m$  and  $\gamma$  parameterize a minimum subsistence amount of consumption and risk aversion, and  $\rho$  is the subjective discount rate. HARA preferences, capturing decreasing relative risk aversion when  $c_m > 0$ , allows us to reconcile the theoretical predictions that, under decreasing relative risk aversion, evasion demand increases with income (Allingham and Sandmo, 1972; Slemrod and Yitzhaki, 2002; Guyton et al., 2023) with the empirical evidence that evasion rates are higher among the high-income individuals (Johns and Slemrod, 2010; Alstadsæter et al., 2019; Londoño-Vélez and Ávila Mahecha, 2021; Leenders et al., 2023).

Entrepreneurial agents use their capital endowment  $k_t$  to produce output (income) given their productivity  $a_t$  following:<sup>3</sup>:

$$y_t = a_t k_t, \quad (2)$$

where  $a_t \in (\underline{a}, \bar{a})$  may be modeled by a stochastic random process or kept fixed throughout

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<sup>3</sup>In line with previous models that incorporate heterogeneous agents and idiosyncratic production risk (Angeletos, 2007).



the agents' life. In this section, we assume the latter scenario which allows for closed-form solutions of agents' optimal choices. However, in the numerical simulations we will assume households' productivity evolves according to the following Cox-Ingersoll-Ross (CIR) process:

$$da_t = \mu (\tilde{a} - a_t) dt + \sigma \sqrt{a_t} dZ_t, \quad (3)$$

where  $Z_t$  is a standard Brownian motion.  $\mu$  parametrizes the speed at which the productivity  $a_t$  reverts to its long-term level  $\tilde{a}$  and  $\sigma$  captures its volatility.

Similar to Tella (2017), there is a complete financial market in which agents can exchange claims written on  $Z_t$  and earn the (exogenous) risk premium  $\pi$ . Agents can allocate a fraction  $\theta_t \in (0, 1)$  of their capital holdings to these claims to hedge their income risk.

The choice of this setting implies that all types of income are taxed equally and does not distinguish if this income is paid in the form of wages, dividends, or profits. As most unreported income stems from self-reported income from entrepreneurs (Johns and Slemrod, 2010; Kleven et al., 2011; Di Nola et al., 2021)<sup>4</sup>, we believe that a setting where future income streams are driven by capital accumulation through risky investments, rather than by increases in labor efforts, are better suited to understand what drives the heterogeneity of tax evasion behavior among entrepreneurial tax payers.

## 2.2 Taxes and Evasion

The government levies linear income taxes at the constant  $\tau \in [0, 1]$  to finance public spending  $g_t$ . As in Gamannossi degl'Innocenti et al. (2022), taxpayers do not consider public spending in their decision, as they do not internalize the link between tax revenues and public good provision (i.e., they are subject to fiscal illusion). As a result of these assumptions, taxpayers' capital holdings with perfect tax compliance evolve with dynamics:

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<sup>4</sup>In the US, wage misreporting rates contributed to about 1% of its total Gross Tax Gap from 2014-2016 (IRS, 2019).

$$dk_t = [a_t k_t (1 - \tau) + \theta_t k_t \pi - c_t] dt + \theta_t k_t \sigma \sqrt{a_t} dZ_t. \quad (4)$$

**Simple and Sophisticated Evasion:** Taxpayers can minimize their tax liabilities ( $y_t \cdot \tau$ ) by either using a simple strategy to evade a fraction  $e_t \in (0, 1)$  of their taxes or by adopting a more sophisticated strategy that allows them to evade a share  $\nu_t \in (0, 1)$  of their tax burden, where  $0 \leq e_t + \nu_t \leq 1$ .

In this setting, simple evasion  $e_t$  can be interpreted as taxpayers' choice to knowingly conceal, misreport, and or misvalue assets that they know will be punishable if detected in an audit. On the other hand, sophisticated evasion  $\nu_t$  is the taxpayers' choice to limit their tax remittances by re-structuring their assets through the use of complex and opaque strategies created with the explicit intention to exploit the tax code's vulnerabilities, as it is unclear that the use of this strategy will be punishable if detected in an audit.

Similar to Lin and Yang (2001); Gamannossi degl'Innocenti et al. (2022); Levaggi and Menoncin (2016), engaging in simple tax evasion  $e_t$  is costless. Conversely, in line with previous studies (Allingham and Sandmo, 1972; Slemrod and Yitzhaki, 2002; Yitzhaki, 1987; Jakobsen et al., 2019), sophisticated evasion  $\nu_t$  is expensive. Formally, an agent who chooses to use sophisticated strategy  $\nu_t$  faces the following costs:

$$f(\nu_t) = \chi_0 \nu_t + \frac{\chi_1}{2} \nu_t^2 \quad (5)$$

per unit of capital  $k_t$ , where  $\chi_0$  and  $\chi_1$  are two positive constants.

Empirically, the parameter  $\chi_0$  represents the initial/minimum deposits and payments required to open a financial vehicle to re-arrange the taxpayers' income structure<sup>5</sup>.  $\chi_1$  parametrizes the variable costs required to rescale these structures.

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<sup>5</sup>(E.g., the minimum amount of money required to open an offshore account in a tax haven or the cost to hire the legal and account expertise necessary to conceal part of their income through pass-through businesses (such as S corporations and partnerships in the US))

### 2.2.1 Expected Evasion Costs:

In line with past models following Allingham and Sandmo (1972), tax evasion demand is strongly determined by the expected costs of being audited and penalized if caught evading.

To this end, we assume the following:

**No Voluntary Compliance Incentives:** Tax payers' utility function and subsequent evasion choices are not driven by a sense of civic duty, intrinsic motivation, and/or social norms.<sup>6</sup>

**Identical Random Auditing Processes:** Tax payers expect both simple and sophisticated forms of evasion  $(e_t, \nu_t)$  are subject to random audits by the government with the same probability. Auditing events follow a Poisson process  $\Pi_t$  with instantaneous intensity  $\mathbb{E}_t[d\Pi_t] = 1 - e^{-\lambda dt} \approx \lambda dt$ .

**Simple Evasion is always fined:** Conditional on being subject to a random audit, simple evasion  $e_t$  will always incur a fine  $\eta$  proportional to the amount of tax liabilities attempted to evade:  $\eta \cdot e_t \cdot y_t \tau$

Both forms of tax evasion are subject to random audits by the government. After being subjected to a random audit, the government imposes a fine whose magnitude depends on the total amount of evasion considered illegal by the government. More specifically, random audits that detect simple tax evasion  $e_t$  always incur a fine  $\eta$  proportional to the total amount of tax liabilities  $y_t \cdot \tau$ . On the other hand, sophisticated evasion  $(\nu_t)$  may be considered legal avoidance by the government *after* with probability  $\beta$  and, as a result, is not be fined.

In summary, implementing the evasion strategy  $(e_t, \nu_t)$  entails the following exposure to auditing events:

$$\eta (e_t + (1 - \mathbb{I}_{e_t \neq \nu_t}) \nu_t) \tau a_t k_t d\Pi_t,$$

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<sup>6</sup>See Luttmer and Singhal (2014) for a further discussion on the relevance of this assumption.

where  $\mathbb{I}_x$  is the indicator function taking value one when event  $x$  is true. Therefore, the expected fine upon auditing equals

$$\eta(e_t + (1 - \beta)v_t)\tau a_t k_t \lambda \cdot dt \quad (6)$$

where we have used the expected value of the indicator function equals the probability of its argument ( $\mathbb{E}_t[\mathbb{I}_{e_t \neq v_t}] = \beta$ ).

When taking tax evasion into account, the agents' capital holdings evolve according to the following stochastic differential equations

$$\begin{aligned} \frac{dk_t}{k_t} = & \left[ a_t(1 - \tau(1 - e_t - v_t)) + \theta_t \pi - f(v_t) - \frac{c_t}{k_t} \right] dt + \\ & + \theta_t \sqrt{a_t} \sigma dZ_t - a_t \eta(e_t + (1 - \mathbb{I}_{e_t \neq v_t})v_t) \tau d\Pi_t. \end{aligned} \quad (7)$$

**Fine Enforcement Capacity  $\beta$ :** Following Gamannossi degl'Innocenti et al. (2022), we can interpret the parameter  $\beta$  as the fine enforcement capacity of the government, capturing the following features: (i) the simplicity of the tax code, (ii) the resources tax authorities have available, and (iii) the efficacy of courts. However, unlike their setting, we consider avoidance an outcome of the sophisticated evasion audit when  $\beta = 1$  rather than an optimal tax-minimizing strategy by the agent. Figure 1 below illustrates the sequence of events and the possible outcomes given taxpayer's evasion decisions:

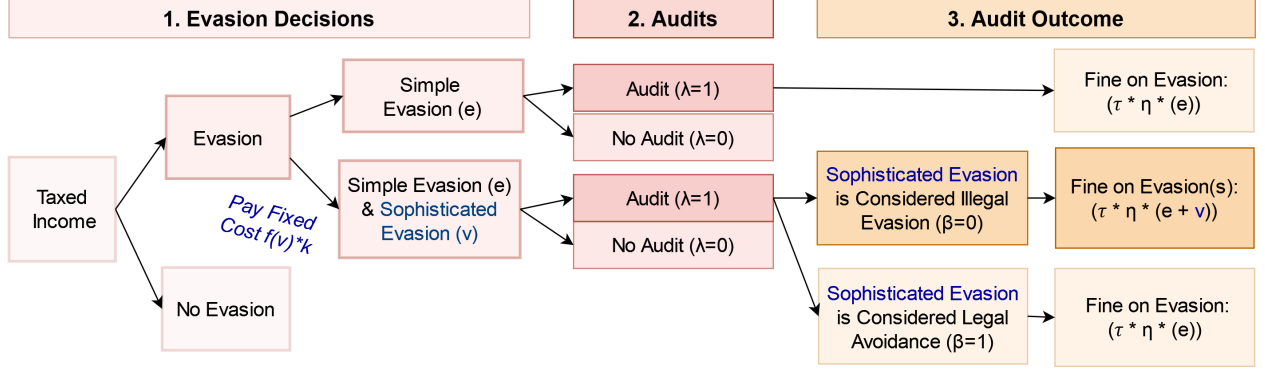


Figure 1: **Flowchart of Taxpayer Evasion Decisions and Audit Outcomes**

This flowchart illustrates the sequential process of taxpayer decisions regarding evasion (Step 1), the audit process (Step 2), and the corresponding audit outcomes (Step 3). It highlights the distinction between simple and sophisticated evasion strategies, the role of audits in detecting evasion, and how enforcement parameters influence the classification of sophisticated evasion as either illegal or legal avoidance.

In this setting, tax payers' expected evasion costs are determined by their perception on how effectively will the government leverage the information acquired by an audit  $\lambda$  to effectively fine them if they choose to evade in a sophisticated manner. Incorporating  $\beta$  allows us to distinguish the effect an increase in the expected consequences of a normal fine would have on agent's evasion behavior compared to an increase in the expectation that the government can successfully use that information to fine them. This distinction will be shown to be especially important for richer tax payers - as any increase in the government's capacity to detect evasion, by increasing the chances they will be randomly audited, may have a much more diminished impact in deterring their evasion choices as long as they expect to be able to afford sophisticated evasion in the future.

Consequently, legal avoidance is an outcome, not a choice made by taxpayers. When  $\beta = 1$ , no fine is imposed and fraction  $\nu \cdot y_t \tau$  is considered legal avoidance. This outcome is critical in modeling taxpayers' optimal evasion decisions across the income distribution and evaluating their aggregate impact on government revenues. From the taxpayers' perspective,  $\beta$  is an institutional parameter that reflects how capable governments are able to impose a fine on their evasion behavior conditional on them being audited. The closer  $\beta$  is to 1, the greater amount of their income can be considered as possible legal avoidance rather than

illegal evasion when audited. In other words  $\beta$  directly influences the size of the "gray area" between illegal evasion and legal avoidance a tax payer can abuse.

## 2.3 Taxpayer's problem

Formally, each taxpayer chooses its consumption  $(c_t)$ , evasion  $(e_t, \nu_t)$ , and risk-taking  $(\theta_t)$  strategy to maximize (2.1) subject to (7),  $k_t > 0$ , and  $\nu_t \geq 0$ , as summarized in the following:

$$\begin{aligned} & \max_{\{c_t, \theta_t, e_t, \nu_t\}_{t \in [t_0, \infty)}} \mathbb{E}_0 \left[ \int_0^\infty e^{-(\rho+\phi)t} \left( \frac{(c_t - c_m)^{1-\gamma}}{1-\gamma} + \phi_\chi \right) dt \right] \\ & \text{s.t.} \\ & dk_t = [a_t k_t - a_t k_t \tau (1 - e_t - \nu_t) + \theta_t k_t \pi - f(\nu_t) k_t - c_t] dt, \\ & \quad + \theta_t k_t \sigma_{a_t} dZ_{a_t} - \eta (e_t + (1 - \mathbb{I}_{e \neq \nu}) \nu_t) \tau a_t k_t d\Pi_t, \\ & \nu_t \geq 0 \end{aligned} \tag{8}$$

As we show in Appendix A, interior solutions of this problem's HJBE form solved with a guess and verify function  $V(k, a) = \frac{F(a)}{1-\gamma} (k - H(a))^{1-\gamma}$  yield:

$$c_t^*(k_t, a_t) = c_m + F(a)^{-\frac{1}{\gamma}} (k_t - H(a)), \tag{9}$$

$$\theta_t^*(k_t, a_t) = \frac{k_t - H(a)}{k} \frac{\pi}{\gamma \sigma^2} \tag{10}$$

$$e_t^*(k_t, a_t) = \frac{(k_t - H(a))}{a_t k_t \tau \eta} \left( 1 - (\eta \lambda)^{\frac{1}{\gamma}} \right) - (1 - \beta) v_t^*, \tag{11}$$

$$v_t^*(a) = \max \left\{ \frac{\tau a_t \beta - \chi_0}{\chi_1}, 0 \right\} \tag{12}$$

where level function  $H(a)$  is pinned down by

$$H(a) = \frac{c_m}{a_t (1 - \tau + \tau \beta v^*(a)) - f(v^*(a))}. \tag{13}$$

and  $F(a)$  satisfies

$$F(a)^{-1/\gamma} = \frac{1}{\gamma} \left[ \rho + \lambda + \phi(1 - \chi) - \lambda(\eta\lambda)^{\frac{1-\gamma}{\gamma}} \right] + \frac{\gamma - 1}{\gamma} \left\{ a_t(1 - \tau + \tau\beta v^*(a)) - f(v^*(a)) + \frac{1}{\eta} [1 - (\eta\lambda)^{1/\gamma}] + \frac{\pi^2}{2\gamma\sigma^2} \right\}. \quad (17)$$

The optimal consumption  $c_t$  and risk-taking  $\theta_t$  strategies derived are consistent with the ones obtained from a standard consumption/asset-portfolio problem under HARA preferences<sup>7</sup>.

Simple tax evasion ( $e_t^*$ ) increases with capital  $k_t$  and decreases directly with productivity  $a_t$ . Its overall relationship with  $a_t$ , however, is ambiguous, as it depends on the slope of  $H(a)$ . Conditional on being positive, the optimal level of sophisticated evasion  $\nu_t^*$  also increases directly with productivity. Moreover, it rises with higher taxes ( $\tau$ ) and decreases with greater enforcement capacity ( $\beta$ ). Perhaps surprisingly,  $\nu_t^*$  is not directly affected by simple evasion deterrence parameters, such as the audit probability ( $\lambda$ ) and fines ( $\eta$ ). Notably, these parameters influence  $\nu_t^*$  only if its productivity is large enough. Productivity  $a_t$  is key in determining both the total evasion demand. Unlike, Menoncin et al. (2022), total evasion demand strongly depends on  $a_t$  over time. This observation is key as total evasion varies both by time and the productivity distribution of  $f(a_t)$ .

Consequently, the optimal strategies in (12) and (11) reveal that simple and sophisticated evasion are substitutes - mirroring a documented empirical phenomenon seen in studies such as in Guyton et al. (2023); EU Tax Observatory (2023); Londoño-Vélez and Ávila Mahecha (2021); Leenders et al. (2023).

## 2.4 Government and Fiscal Revenue Collection Capacity

The government does not observe the agents' true income  $y_t$  and generates instantaneous revenue  $T_t$  through direct tax remittances and by enforcing fines on evaders who have been

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<sup>7</sup>See Merton (1969).

audited.

Formally, for an individual taxpayer with income  $y_t = a_t k_t$  and a given set of evasion strategies  $(e_t^*, \nu_t^*)$ , the government's total expected total revenues from the agent can be formulated as:

$$\mathbb{E}_t[T_t] = \mathbb{E}_t \left[ \underbrace{k_t a_t \tau (1 - e_t^* - \nu_t^*)}_{\text{Direct Revenues} := G_t^{\text{Gross}}} dt + \underbrace{k_t a_t \tau \eta (e_t^* + (1 - \mathbb{I}_{e_t \neq v_t}) \nu_t^*)}_{\text{Fine Revenues} := \bar{\eta}_t} d\Pi_t \right] \quad (14)$$

We can note that the gross tax gap is equal to the total amount of taxpayer's evasion, where  $G_t^{\text{Gross}} = 1 - e_t^* - \nu_t^*$ . We can conveniently re-arrange 14 to express the government's expected *Net Tax Gap*,  $G_t^{\text{Net}}$  as as the share of revenues collected after enforcement of fines and auditing occurs, as the share of all tax liabilities:

$$\mathbb{E}_t[G_t^{\text{Net}}] = \mathbb{E}_t \left[ \frac{G_t^{\text{Gross}} - \bar{\eta}_t}{k_t a_t \tau} \right]. \quad (15)$$

We then plug in agents' optimal evasion choices  $e_t^*$  and  $\nu_t^*$  into Eq.(15) as:

$$E_t[G_t^{\text{net}}] = [\beta \cdot \max \left\{ \frac{\tau a_t \beta - \chi_0}{\chi_1}, 0 \right\} - \frac{(k_t - H(a))}{k_t a_t \eta} (1 - \eta \lambda) \left( 1 - (\eta \lambda)^{\frac{1}{\gamma}} \right)] dt \quad (16)$$

Let us then define the government's fiscal revenue collection capacity  $FC$  to close the tax gap  $G_t^{\text{net}}$  contributed by an individual as the marginal utility of an increase in policies  $x \in (\tau, -\beta, \lambda, \eta)$  per individual. Thus,

$$FC_X(X; k_t, a_t) :=, \frac{\partial G^{\text{net}}(k_t, a_t)}{\partial X} \quad (17)$$

In the next sections, we show how changes in different policy parameter affects the tax gap conditional on households' income and how the government's fiscal capacity  $FC_X(X; k_t, a_t)$  may be limited by the distribution of households.



## 2.5 Comparative Statics

To understand the effects of a change in any of the government's policy parameters  $X \in (\tau, \beta, \lambda, \eta)$  on taxpayer's optimal evasion choices and on the government's net tax gap function 16, we analyze how an increase in any in the short-run.

**Households:** The impact different policies have on tax payer's total evasion strategies vary on their income level. Table 1 summarizes how individual taxpayer's optimal evasion strategies react to a change in these policies by analyzing  $e_t^*, \nu_t^*$  and  $\bar{E}_t = e_t^* + \nu_t^*$ 's derivative with respect to an increase in  $(\eta, \lambda, \tau, \beta)$ . However, to understand how the changes in policies unevenly affect taxpayer's total evasion strategy  $\bar{E}$  depending on their level of income  $y_t$ , we take the second derivative of  $\bar{E}$  with respect to their income  $y_t$ . The derivations for these are detailed in the numerical appendix (6) and are shown under the  $\frac{\partial \bar{E}}{\partial y \partial X}$ .

In this setting, enforcement instruments  $(\lambda, \eta)$  always help close the tax gap by deterring simple evasion in the short run, while tax-rate hikes and broader avoidance windows lower it. However, the size and magnitude of the policy parameter's effect on agent total evasion behavior may change depending if they can engage in sophisticated evasion  $\nu_t^*$  or not. Taxpayer's total evasion behavior,  $\bar{E}$ , increases with their net worth  $k_t$  and income  $y_t$ , as  $\frac{\partial(\bar{E}_t)}{\partial y} > 0$ .<sup>8</sup>—

**Sophisticated Evasion's Threshold:** Additionally, the threshold where agents can start engaging in sophisticated evasion:

$$\tau a \beta \geq \chi_0 \quad (18)$$

plays an essential role in the total evasion dynamics of taxpayers, exacerbating agents' responses to market or policy changes. Once crossed, it decreases the efficacy of enforcement policies  $\lambda, \eta$  even further, the higher the income  $y_t$  or capital  $k_t$  the taxpayer has. In the

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<sup>8</sup>Under this setting, this holds true even when we assume the minimum consumption level  $c_m = 0$ , as  $H(a) \approx \frac{c_m}{a_t(1-\tau) + \frac{(\tau a \beta - \phi - \theta_0)^2}{2\theta_1}}$

short run, as individual tax payer's capital stock  $k_t$  grows through stochastic changes in their productivity  $a_t$  and  $\theta_t$  investments in risky capital, the deterrence capacity of random audits  $\lambda$  in reducing total evasion is strongly diminished at the threshold where agents can afford to pay  $\chi_0$ , and exacerbated by lower variable costs  $\chi_1$ .

Table 1: **Short-run effect of enforcement/policy parameters on individual tax-payer's optimal evasion choices.**

This table shows the sign of the derivatives of the function in the column with respect to the parameter in the row, as detailed in this section. Column "Threshold Amplification?" denotes if a change in parameter X has a disproportionately large impact on tax payers' total Evasion  $\bar{E}$  past threshold  $\tau a\beta \geq \chi_0$ .

Parameter (X)	$\nu_t^*$	$e_t^*$	$\bar{E} = \nu_t^* + e_t^*$	$\frac{\partial \bar{E}}{\partial y_t \partial X}$	Threshold Amplification?
$\eta$	0	—	—	—	No
$\lambda$	0	—	—	—	No
$\beta$	+	<i>und.</i>	<i>und.</i>	+	Yes
$\tau$	+	<i>und.</i>	<i>und.</i>	+	Yes

This illustrates the importance enforcement  $\beta$  in reducing the tax gap contributions of richer tax payers, relative to tax payer's capital accumulation process. This does not necessarily mean that in the long run audits and fines are completely inefficient in reducing total evasion. Governments' deterrence capacity will depend on the drivers of agents' capital accumulation process - such as their initial productivity  $a_0$ , the risk premium of  $\pi$  and volatility  $\sigma_{at}$  - and on the government's capacity to distinguish simple from sophisticated evasion;  $\beta$ .

**Government: Revenues and the Tax Gap** In the short-run, higher levels of income  $y_t = a_t \cdot k_t$  lead to diminished efficacy of random audits and fines,  $\lambda, \eta$ , in reducing the the net-tax gap, as  $\frac{\partial dG_t^{net}}{\partial \lambda \partial y} > 0$  and  $\frac{\partial dG_t^{net}}{\partial \lambda \partial y} < 0$ , while the opposite happens for increased enforcement  $\beta$ . This effect is exacerbated for households above threshold 18. Conversely, higher

enforcement  $\beta$  and taxes  $\tau$  depend on  $a$  and  $k$ . For these reasons, we can summarize the short-run effects in table 2 below and later corroborate them with our numerical simulations.

Table 2: **Short-run effects of enforcement/policy parameters on the Net Tax Gap.**

Parameter (X)	Net Tax Gap ( $G_t^{net}$ )	Sign Depends $\nu(a) > 0?$	$\frac{\partial FC(y_t)}{\partial y_t \partial X}$
$\eta$	—	No	—
$\lambda$	—	No	—
$\beta$	+	Yes	+
$\tau$	und.	Yes	und.

This table shows the signs of the derivatives of the Net Tax Gap function, along with a column indicating if the direction of the effect depends on agents' ability to engage in sophisticated evasion  $\nu^* > 0$ . Column  $\frac{\partial FC(y_t)}{\partial y_t \partial X}$  summarizes if fiscal capacity of government to collect revenues and close tax gap decreases with higher income  $y_t$

**Uneven Enforcement Effects across the Income Distribution.** Given the discontinuity present in Eq. 12 due to  $\nu_t^*$ , we must analyze the impact of a change in a policy parameter for the amount of tax payers where threshold Eq. 18 holds or not. Higher  $\beta$ , or a decrease in the enforcement capacity of governments, leads to a substitution effect between sophisticated evasion  $\nu_t$  and simple evasion  $e_t$ . For agents who cannot afford the fixed cost anymore, they engage in higher simple evasion  $e_t$  instead which can be more effectively reduced by other enforcement parameters  $\eta, \lambda$ . However, it will have no effect on agents below the threshold 18.

Nonetheless, enforcement instruments  $\lambda, \eta$  raise fiscal capacity in the short run; tax-rate hikes and broader avoidance windows ( $\uparrow \beta$ ) lower it. However, while the expected sign change of as change in random audits and fines are always found to be positive and do not depend on the initial distribution - their magnitude may be impacted by it.

## 2.6 Stationarity and Aggregation

While the full stationary distribution can be obtained numerically, most analytical results rely on the assumption that the *excess capital* variable,

$$z_t := k_t - H(a), \tag{19}$$

is stationary in expectation. The function  $H(a)$  denotes the subsistence-adjusted wealth term derived under HARA preferences, capturing the level of assets necessary to sustain future minimum consumption. Consequently,  $z_t > 0$  measures the disposable wealth available for risk-taking and potential tax evasion once agents have secured the subsistence level of wealth. This transformation allows all policy functions to be expressed in terms of productivity  $a$  alone and greatly simplifies the analytical characterization of household behaviour.

**Stationarity assumptions.** We maintain the following regularity conditions:

**Assumption 1: Positive Excess Capital** The excess wealth variable  $z_t = k_t - H(a)$  is strictly positive and constant in expectation.

**Assumption 2: Policy parameters** Policy parameters satisfy

$$\tau, \beta, \lambda \in (0, 1), \quad \gamma, \eta > 1, \quad c_m, \chi_0, \chi_1 > 0.$$

Assumptions 1–2 guarantee that all interior solutions for consumption and evasion are well defined and that all agents hold positive disposable wealth. Combined with the Poisson death-and-rebirth process, these conditions imply the existence of a stationary cross-sectional density  $p(a)$  of productivity types and a stationary distribution of excess wealth  $p(z|a)$ .<sup>9</sup>

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<sup>9</sup>The full derivation is provided in Appendix X.

**Type-specific stationarity.** The existence of a stationary distribution of excess capital requires that, for each productivity type  $a$ , the expected drift in  $z_t$  is zero,  $\mu(a) = 0$ , where

$$\mu(a) = \frac{1}{\gamma} \left[ a(1 - \tau + \tau\beta v^*(a)) - f(v^*(a)) + \frac{1}{\eta} - \rho - \lambda - \phi \right] + \lambda((\eta\lambda)^{1/\gamma} - 1). \quad (20)$$

Stationarity thus requires that, in expectation, the after-tax and after-enforcement return on wealth equals the effective discount rate and expected audit loss. When  $\mu(a) > 0$ , excess wealth accumulates on average, leading to persistent asset growth for type  $a$ ; when  $\mu(a) < 0$ , excess wealth declines toward the subsistence threshold  $H(a)$ .

**Threshold productivity.** Substituting the optimal sophisticated-evasion policy into (20), the zero-drift condition  $\mu(a) = 0$  becomes

$$a(1 - \tau + \tau\beta v^*(a)) - f(v^*(a)) = \rho + \lambda + \phi_\chi - \frac{1}{\eta} - \gamma\lambda((\eta\lambda)^{1/\gamma} - 1), \quad (21)$$

where  $\chi$  captures the survival-adjusted probability of remaining in the economy.

For types below the sophisticated-evasion threshold ( $\tau a\beta \leq \chi_0$ ), we have  $v^*(a) = 0$ , and the stationarity condition simplifies to

$$a(1 - \tau) = K, \quad K := \rho + \lambda + \phi_\chi - \frac{1}{\eta} - \gamma\lambda((\eta\lambda)^{1/\gamma} - 1). \quad (22)$$

For types above the threshold ( $\tau a\beta > \chi_0$ ), substituting  $v^*(a) = (\tau a\beta - \chi_0)/\chi_1$  yields

$$a(1 - \tau) + \frac{1}{2\chi_1}(\tau a\beta - \chi_0)^2 = K. \quad (23)$$

Equation (23) is strictly increasing in  $a$ , implying the existence of a unique productivity threshold  $\bar{a}$  satisfying  $\mu(\bar{a}) = 0$ . Agents with  $a > \bar{a}$  experience positive wealth drift ( $\mu(a) > 0$ ), while those with  $a < \bar{a}$  converge to the subsistence level ( $\mu(a) < 0$ ). This threshold divides the population into accumulating and decumulating types, determining the shape of

the stationary distribution and the share of high-productivity evaders.

**Aggregation** Aggregate income  $Y_t$  and government revenues  $T_t$  can be obtained by integrating individual decisions over the stationary density  $p(z, a)$ .

$$E_t[Y_t] = \int_{\underline{a}}^{\bar{a}} \int_0^\infty E_t[a \cdot (H(a) + z_t)] \cdot f(z, a) \cdot dz \cdot da \quad (24)$$

$$E_t[T_t] = \int_{\underline{a}}^{\bar{a}} \int_0^\infty E_t \left[ k_t a_t \left( 1 - \beta \cdot \max \left\{ \frac{\tau a_t \beta - \chi_0}{\chi_1}, 0 \right\} \right) - z_t (1 - \eta \lambda) \left( 1 - (\eta \lambda)^{\frac{1}{\gamma}} \right) \right] \cdot f(z, a) \cdot dz \cdot da \quad (25)$$

In the numerical simulations, aggregate quantities can therefore be simulated by drawing from  $p(a)$  and computing ergodic averages, as discussed in Section 4. Monte Carlo simulations converge through ergodicity, ensuring a stable invariant distribution of productivity and excess capital.

## 2.7 The Dispersion Channel

The effect of inequality on fiscal revenue collection capacity can be examined by varying the dispersion of productivity,  $\sigma_a$ , which controls the spread of the stationary distribution  $p(z, a)$ . An increase in  $\sigma_a$  reallocates mass toward high-productivity agents, raising the share of taxpayers for whom the sophisticated-evasion margin  $v^*(a) > 0$  is active.

**Dispersion Channel:** Holding the policy vector  $X = \{\tau, \lambda, \eta, \beta\}$  fixed, an increase in income dispersion  $\sigma_a$  reduces fiscal capacity:

$$\frac{\partial FC(X, \sigma_a)}{\partial \sigma_a} < 0.$$

as long as there exists agents whose productivity level  $a \geq \frac{\chi_0}{\tau \beta}$  allows them to engage in sophisticated evasion  $\nu(a)^* > 0$ .

**Mechanism.** When  $\sigma_a$  increases, more agents satisfy  $a \geq \frac{\lambda_0}{\tau\beta}$ , activating the sophisticated-evasion channel in (23). These agents face higher returns to wealth accumulation and lower compliance elasticities with respect to enforcement parameters  $(\lambda, \eta)$ . Consequently, aggregate tax liabilities  $\int_{\underline{a}}^{\bar{a}} \int_0^\infty \tau a_t k_t \cdot f(z, a) \cdot dz \cdot da$  become more concentrated among taxpayers who contribute less marginal revenue per unit of enforcement. This *composition effect* weakens the aggregate responsiveness of revenues to policy changes, while a complementary *concentration effect* magnifies the importance of the top of the distribution in total evasion.

**Interpretation.** Together, these forces flatten the fiscal-capacity and Laffer curve: even as average productivity rises, the effective revenue potential declines because the tax base shifts toward high-productivity, low-compliance agents. Section 4 quantifies this mechanism and shows that increasing  $\sigma_a$  from 0.15 to 0.35 lowers maximal fiscal capacity and raises the revenue-maximizing tax rate.

### 3 Numerical Methods and Calibration

This section describes the strategies used to derive this paper’s main results. To do this, we describe the numerical solution methods used, how we use them to validate our results shown in the comparative statics section, and calibration strategies.

#### 3.1 Methodology

All results presented are resolved numerically. To do this, we can resort to numerical simulation methods to endogenously determine the joint stationary  $f(a, k)$  using a set of initial parameters, whose choice is described in our calibration methods below. This allows us to then compute optimal policies  $\{e_t^*(k_t, a_t), v_t^*(k_t, a_t), \theta_t^*(k_t, a_t), c_t^*(k_t, a_t)\}$  and their aggregate effects on the Net Tax Gap  $G_t^{net}$  as well as its components.

Specifically, we first approximate the objects  $F(a, t)$  and  $H(a, t)$  by using Monte Carlo simulations of  $dZ_t$  realizations over a discretized productivity grid  $a_M$ , where  $M = 10$  are

the number of linearly spaced intervals ranging from  $(\underline{a}, \bar{a})$ . Using these objects, we then compute the optimal policy function in each state space point and simulate the dynamics of the controlled state (capital  $k_t$  and productivity  $a$ ) process over a long time horizon;  $T = 25,000$ . we then discretize capital grid  $k_N$ , where  $N = 50$ , spaced according to the computed distribution of  $k$ , with intervals ranging from the minimum and maximum  $k_t$  simulated  $\in (\underline{k}, \bar{k})$ . Under the assumption that the joint dynamics of these processes are ergodic, we use the obtained (empirical) density function to approximate  $f(a, k)$ . By multiplying all optimal policies times the stationary distribution  $f(a, k)$  yields the optimal policies of the continuum of agents across the distribution. Finally, the aggregate results at the stationary distribution are computed by integrating the optimal policies over state space  $f(a, k)$ .

We validate our main long-run results by calibrating the model to match the empirical observations found by Guyton et al. (2023) on the aggregate and distribution of tax evasion by income group and type of evasion seen in the U.S. during 2011-13. The main results are presented in Section 4.3.

We then do policy experiments in this setting. The short-run experiment computes the tax gap by changing optimal policies but keeping the distribution constant; the long-run adjusts both. Then, we compare the outcomes.

## 3.2 Calibration

The following parameters are externally calibrated: The nominal tax rate ( $\tau$ ), the audit intensity ( $\lambda$ ), and the auditing fine ( $\eta$ ). These are consistent with IRS (2019). In particular, the values of  $\tau$  and  $\lambda$  are taken as the average rates across all taxpayers during 2011-13 in the US. The maximum evasion fine,  $\eta$ , is the maximum penalty rate. The risk aversion  $\gamma$  and the discount rate  $\rho$  take standard values from the literature. Table 3 summarizes these parameter values and their sources.

Government's fine enforcement capacity  $\beta$ , the cost parameters  $\chi_0$  and  $\chi_1$ , and the subsistence consumption level  $c_m$  are chosen to match a few key moments on the long-term



Table 3: Externally Calibrated Parameters

Parameter	Description	Value	Source
$\bar{a}$	Long term Aggregate Productivity	1.00	-
$\pi$	Premium of Risky Assets	0.06	Graham and Harvey (2010)
$\lambda$	Average Probability of being Audited	0.03	IRS (2019)
$\eta$	Max. Evasion Fine of 75%	1.75	IRS (2019)
$\tau$	Tax Rate	0.20	IRS (2019)
$\gamma$	Risk Aversion	2.50	Standard
$\rho$	Discount Rate	0.015	Standard

distribution in the steady steady state of tax evasion across income groups, according to Guyton et al. (2023), as well as their total contribution to the tax gap. Table 4 reports the internally calibrated parameters and the corresponding target moments.

Table 4: Internally Calibrated Parameters

Parameter (X)	Description	Value	Moment	Target	Unit
$\mu, \sigma$	Mean-reversion time, productivity risk	(0.13, 0.8)	US GINI Income	0.47	(0-1)
$\beta$	Fine Enforcement Capacity	0.8	Soph. Evasion Contribution to Tax Gap	2	% of all tax liabilities $Y_t\tau$
$\chi_0$	Fixed Cost of Soph. Evasion	0.2	Income Ptile Where Soph. Evasion $> 0$	30	Percentile of Income
$\chi_1$	Variable Costs of Soph. Evasion	0.3	Soph. Evasion by Top 1%	6	% of tax liabilities $Y_t^{\bar{a}}\tau$
$c_m$	Minimum Consumption Level	0.6	Minimum Income Ptile for Simple Evasion	10	Percentile of Income

Where  $Y_t\tau$  denotes all the true tax liabilities summed across all agents and  $Y_t^{\bar{a}}\tau$  is the true tax liabilities of the riches income percentile of income group.

Specifically, we can interpret the long-term productivity  $\tilde{a}$  as the nominal median salary in the U.S. around that that time period, of approximately 50,000\$ (IRS, 2019). This choice of minimum consumption level  $c_m = 0.6$  approximately replicates the ratio of this median income over the minimum wage plus average government transfers estimated at around 27,000\$ nominally where  $\frac{\tilde{c}_m}{\tilde{a}} \approx \frac{27,000}{50,000}$ . This calibration allows for tax payers' to start evading  $e_t > 0$  at the 10th lowest level of income levels we simulate.

Fixed and variable costs  $\chi_0, \chi_1$  are harder to match empirically. Loosely,  $\chi_0$  corresponds to the minimum costs of talking to a tax consultant to re-shuffle one's assets or the minimum deposit required to keep in a haven. There exists a wide range of these minimum estimates in

the literature<sup>10</sup>. However, we target these values to proportionally increase with households net-worth as a fraction of taxpayer’s wealth as fixed costs are  $k_t \cdot f(\nu^*) = k_t \chi_0 \nu_t + k_t \frac{\chi_1}{2} \nu_t^2$ .  $\chi_0$  is chosen to match the percentile of income where sophisticated evasion propensity seen in the income distribution, and  $\chi_1$  is calibrated to match the maximum sophisticated evasion behavior of the richest agents, as a share of all their true tax liabilities, as found by Guyton et al. (2023). Finally,  $\beta$ , the fine enforcement capacity of the government, is set to target the aggregate sophisticated evasion contributions to the Net Tax Gap;  $\int_{\underline{a}}^{\bar{a}} \int_0^\infty \nu_t^* \cdot f(a, k) \cdot da \cdot dk$ .

### 3.3 Thought Experiments

Before presenting the main results, we structure the analysis around a set of thought experiments designed to isolate and evaluate the mechanisms discussed in **Section 2**. These experiments guide the interpretation of the model’s implications across both short- and long-term horizons.

We begin in **Section 4** by validating the model’s predictions against empirical evidence on the U.S. tax gap and the distribution of evasion across income levels. We then conduct short-run comparative statics that assess how changes in enforcement parameters, such as audit probability ( $\lambda$ ), fine rate ( $\eta$ ), tax rate ( $\tau$ ), and fine enforcement capacity ( $\beta$ ), affect evasion decisions and the tax gap while holding the income and capital distribution fixed. These experiments provide a clean benchmark for identifying individual behavioral responses to policy shifts.

In the second part, we simulate the long-run effects of these same policy changes by allowing the distribution of productivity and capital to evolve endogenously. This enables us to assess how enforcement interacts with capital accumulation and evasion behavior over time. Finally, we interpret these results in light of recent and proposed policy initiatives—such as FATCA, CRS, and the U.S. Treasury’s compliance proposals—and discuss their likely effectiveness given the model’s mechanisms.

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<sup>10</sup>These may range from 5,000-100,000\$, where this maximum value stems from the minimum deposit required to declare assets under FATCA (IRS, 2019)

Together, these experiments shed light on both the direct and distributional impacts of enforcement tools and offer practical guidance for designing policies that more effectively reduce the tax gap.

## 4 Quantitative Results

This section presents the main results on the long-term stationary distribution of tax evasion distribution and the resulting aggregate tax gap. We rely on numerical solutions to compute both the stationary distribution  $f(a, k)$  and aggregate outcomes  $G_t^{net}$  and its decomposition. To analyze what effect policies  $\beta, \tau, \eta, \lambda$ , we first establish a baseline model of the long-term stationary distribution of tax evasion and validate its accuracy with the empirical regularities seen in the data.

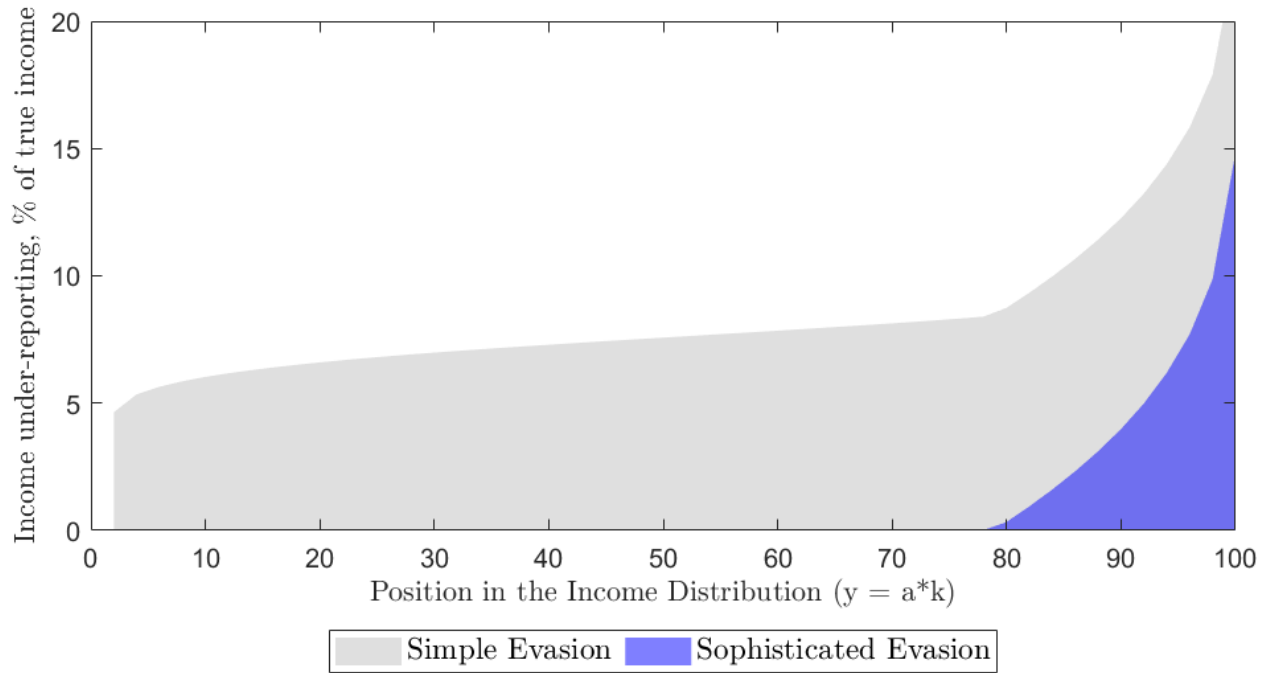
### 4.1 Benchmark and Validation: US Tax Gap

We first simulate the long-term stationary distribution of tax evasion and the resulting aggregate tax gap in the US found by Guyton et al. (2023) using our internal calibration strategy described in the previous section. These benchmark numerical results closely resemble their estimates. Table 5 contrasts the simulated aggregate results with their findings, while figure 2 shows the model’s simulated distribution of total simple  $e_t$  and sophisticated  $\nu_t$  evasion across the income  $y_t = a_t \cdot k_t$  distribution that correspond to the aggregate results shown in 5. Additionally, figure 9 in the appendix compares these results on the distribution with Guyton et al. (2023)’s estimates.

Table 5: **Model Results: Untargeted Moments compared to previous Estimates.**  
Decomposition of aggregate tax gap moments, expressed as percentages of sum of all tax liabilities.

Parameter	Description	Model	Data	Source
Gross Tax Gap	Taxes paid, as % of all Tax Liabilities	11.1	12.8	Guyton et al. (2023)
Net Tax Gap	Gross Tax Gap - Enforcement Revenues, as % of all Tax Liabilities	10.6	11.3	Guyton et al. (2023)
Enforcement Revenues	Revenues from evasion fines, as % of all Tax Liabilities	0.5	1.5	IRS
Total Evasion Rate by Rich	Total Evasion by Richest Income Group, as % of their Income	23.9	21.4	Guyton et al. (2023)

Figure 2: **Model Estimates of U.S. Unreported Income by Percentile of Income, Average 2006-13 (as % of True Income  $y_t$ ).**



This figure plots the models simulated estimates of tax payers’ optimal under-reported income by evasion strategy, optimal evasion type ( $e_t^*, \nu_t^*$ ) and over their position in the simulated stationary income distribution.

The simulated results closely resemble those found by Guyton et al. (2023) as both total evasion rates and contributions to the total net tax-gap increase at higher income levels. The inflection point seen in the total evasion behavior around the 90th percentile of agents shows how the inclusion of an alternative, but costly, to simple evasion  $e_t^*$  drastically increases the

total evasion of behavior of richest tax payers. It is important to note that this inflection point, unlike threshold  $\beta a \tau \geq \chi_0$ , occurs as sophisticated evasion gains dominate simple evasion gains as  $k_t - c_m \cdot H(a, t)^{-1}$  grows at a much faster rate as  $c_m$  becomes insignificant compared to total  $k_t$ . This result is in line with the past theoretical predictions: richer tax payer’s lower risk aversion induces higher levels of total evasion as their demand for both risky assets, in the form of  $\theta_t$  and evaded assets subject to audits  $\lambda$ , increases linearly with their net worth  $k_t$ .

## 4.2 Short-Run

We test how a 1% increase in policy parameter  $X \in (\eta, \lambda, \tau, \beta)$  would affect the total evasion and net tax gap aggregate estimates compared to the baseline simulated distribution  $f(a, k)$  computed for the benchmark US tax gap in the section above. Table 6 summarizes such results<sup>11</sup>.

Table 6: **Short-run effect of enforcement/policy parameters on Aggregate Values (as % increase)**. This table shows the simulated impact of a 1% increase in any of policy parameters on the aggregate tax gap as well its components, assuming a constant distribution  $f(a, k)$  as displayed in the previous section.

Parameter	Simple Evasion $e^*$	Soph. Evasion $\nu^*$	Fine Revenues $\bar{\eta}$	Gross Tax Gap $G^{\text{Gross}}$	Net Tax Gap $G^{\text{Net}}$
$\eta$	-0.149	0.000	-0.003	-0.152	-0.149
$\lambda$	-0.057	0.000	0.002	-0.055	-0.057
$1 - \beta$	0.000	-0.594	0.000	-0.494	-0.494
$\tau$	0.052	0.482	0.003	0.441	0.438

This table corroborates most of the short-term analytical derivations and predictions shown in Section 2. However, under this baseline scenario, an increase in fine enforcement  $1 - \beta$  leads to the greatest decrease in the net tax gap. This is driven by both the large number of agents who choose sophisticated evasion  $\nu_t^* > 0$  and due to the large part of income and its corresponding tax liabilities being concentrated in higher levels of income distribution.

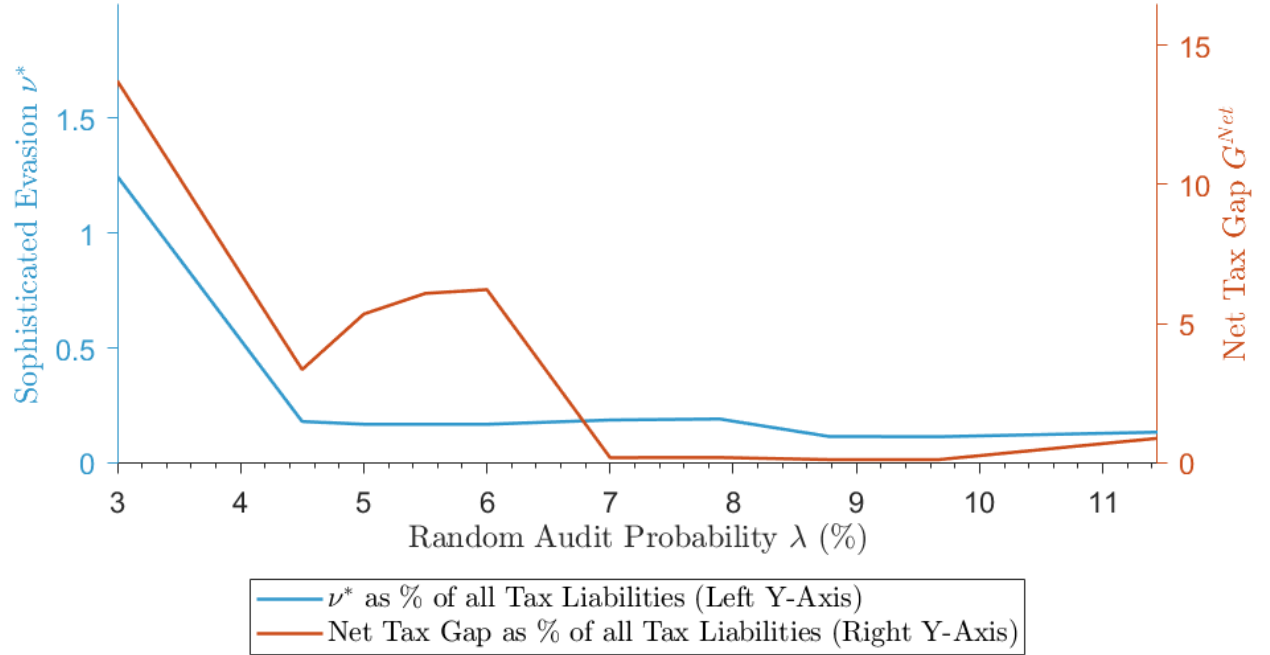
<sup>11</sup>An increase in fine enforcement would be a decrease in  $\beta$ , thus we present the results of and increase in  $1 - \beta$  as it is equivalent.

### 4.3 Long-Term

The aggregate effect of changes in policy parameters  $\eta, \lambda, \tau, \beta$  in the tax-gap and on total evasion in the long-run are not as clear as in the short-run as stationary distribution  $f(a, k)$  may change.

To show this, we can simulate the aggregate net tax gap and total sophisticated evasion  $\nu_t^*$  for different values of auditing  $\lambda$  and enforcement  $\beta$ , holding them constant.

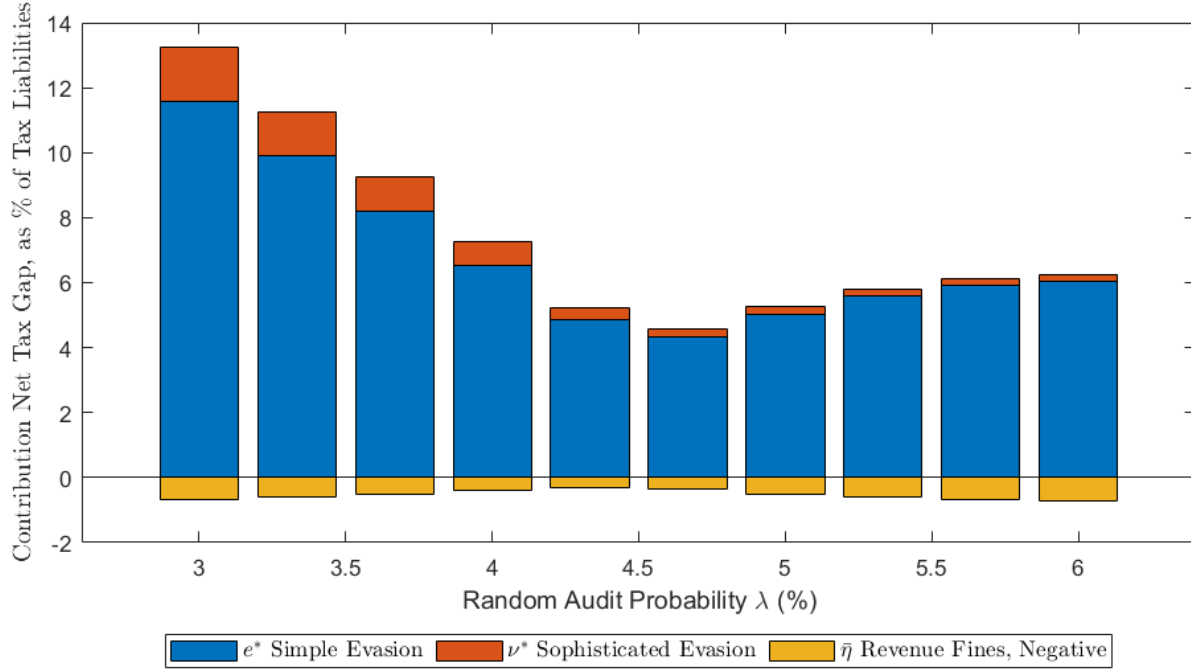
Figure 3: **Aggregate Sophisticated Evasion and Net Tax Gap across all Random Audit Probability in the Long Term** as % of all Tax Liabilities ( $\beta = 0.8$ )



As evidenced in figure 3, an increase in random audits does not necessarily lead to a decrease in the total net tax gap, contrary to the effect it has in the short run (see Section 2). This non-linear relationship between aggregate sophisticated avoidance  $\nu^*$  and the tax gap reflects changes in several aggregate evasion responses. As expected, low levels of random audits induce higher levels of total tax evasion, increasing the tax gap. An increase in the probability of being audited may initially decrease the total tax gap as simple evasion is

deterred, but may its marginal effectiveness declines drastically when several high income tax payers to engage in higher levels of sophisticated evasion. Consequently, the governments' ability to close the tax gap through the use of fines is diminished, as richer agents who engage in  $\nu^*$  are subject to less fines proportional to their income  $y_t$  compared than the others. Its ability to close the tax gap relies on deterring evasion and reducing total gross tax gap  $G^{Gross}$ , rather than making up the potential tax liabilities lost through increased fine revenues  $\bar{\eta}$ . Figure 4 below decomposes the effects increased random audits  $\lambda$  has on closing the tax gap and how revenues from fines are limited in its capacity to reduce the total tax gap.

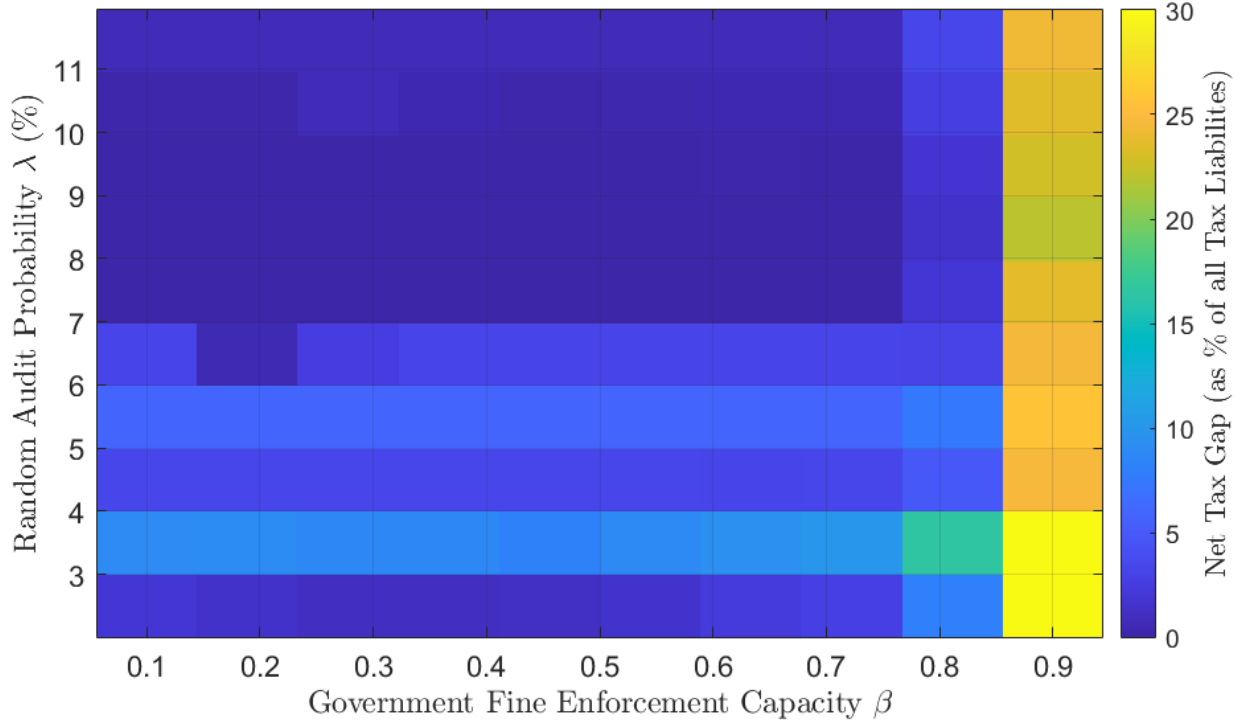
Figure 4: **Decomposition of Aggregate Net Tax Gap** as % of all Tax Liabilities ( $\beta=0.8$ )



This decrease in the ability of random audits decreases as long as  $f(a, k)^{\nu_t^* > 0} > 0$ . This highlights the importance of enforcement  $\beta$  in reducing the tax gap. However, an improvement in fine enforcement quality, or a reduction in  $\beta$ , is only useful in reducing the tax gap if  $f(a, k)^{\nu_t^* > 0} > 0$  in the long-run and may also reduce the total capital accumulation growth of agents above threshold Eq.18. At higher levels of the quality of fine enforcement  $\beta \rightarrow 0$ ,

total sophisticated evasion and the net tax gap remains relatively stable as its driven by the total simple evasion demand across all tax payers. Under very low levels of quality of enforcement  $\beta \rightarrow 1$ , the tax gap increases linearly with total sophisticated  $\nu^*$ , as seen in figure 5.

Figure 5: **Average Long-Term Net Tax Gap estimates across different Random Audit Probability Rates and Government's Fine Enforcement Capacity**



This figure further emphasizes the importance of distinguishing threshold Eq. (18). It can be observed that once  $\beta > 0.8$ , holding all fixed costs and distributional estimates of  $a$  constant, the capacity of random audits to deter the net tax gap is strongly diminished. Even at very high levels of random audits  $\lambda > 10\%$ , tax gaps remain unable to be closed once threshold Eq. (18) is trespassed. This threshold denotes the point where there exists a mass of agents where  $f(a, k)^{\nu_t^* > 0} > 0$ .



## 4.4 Fiscal Capacity and Inequality Dispersion

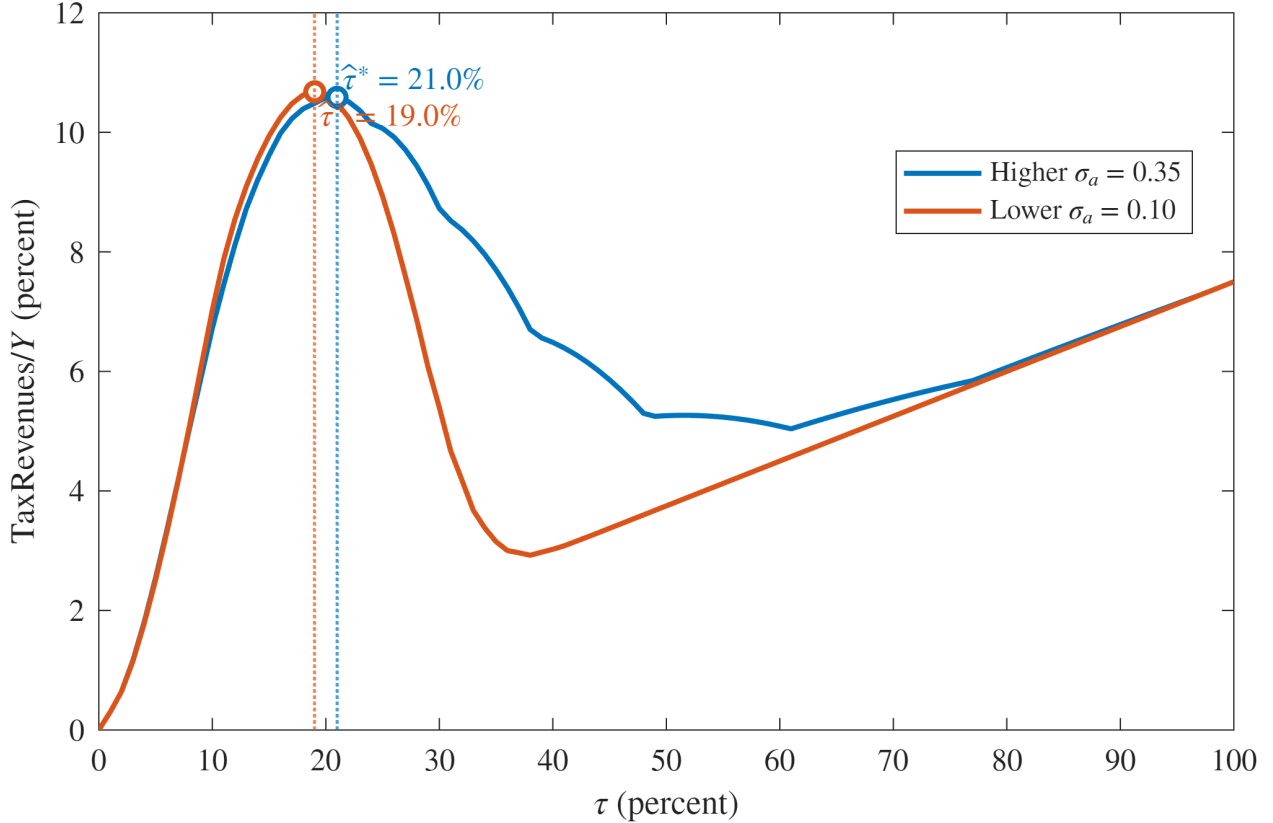
We now examine how changes in income dispersion affect the government’s ability to raise revenues. To do this, we fall back to our analytical results and set stationarity  $z_t$  fixed as detailed in 2. Nonetheless, we hold all other parameters constant but keep  $a$  constant for different households throughout their life. Holding the mean productivity  $\mathbb{E}[a]$  fixed, we increase the variance  $\sigma_a^2$  of the productivity distribution and recompute the stationary equilibrium under the same calibration for  $(\tau, \lambda, \eta, \beta)$ . This exercise isolates the role of heterogeneity in shaping the fiscal–capacity curve.

The aggregate enforcement elasticity, declines mechanically with  $\sigma_a$ , flattening the fiscal–capacity curve because additional enforcement or higher tax rates generate smaller marginal revenue gains. Intuitively, inequality reallocates tax liabilities toward agents whose compliance decisions are less responsive to policy, reducing the slope of the Laffer curve as it lowers the efficiency of each percentage point increase of taxation.

**Inequality and the Laffer Curve:** Figure 6 illustrates how income dispersion affects fiscal capacity in the model. A higher variance of productivity ( $\sigma_a = 0.15$ ) lowers the maximum amount of revenue that the government can raise, even though the revenue-maximizing tax rate shifts slightly to the right. The mechanism is straightforward. When the income distribution becomes more dispersed, a larger share of taxable income is concentrated in households that either earn very little or have access to sophisticated evasion strategies at the top. Both groups contribute less enforceable revenue: low-income households because their tax base is small, and high-income households because they increasingly rely on legally ambiguous forms of evasion that audits and fines cannot effectively deter. As a result, the tax base becomes narrower and harder to enforce, and the government must apply a marginally higher statutory rate to reach the peak of the Laffer curve. However, because the additional tax pressure falls precisely on the segments of the distribution that are least responsive in terms of enforceable revenue, the resulting peak is lower under high dispersion. In short,

greater inequality reduces fiscal capacity not by dramatically altering the optimal tax rate, but by shrinking the portion of income that the government can credibly tax.

Figure 6: **Fiscal capacity: Revenue collecting capacity under different levels of income dispersion.**



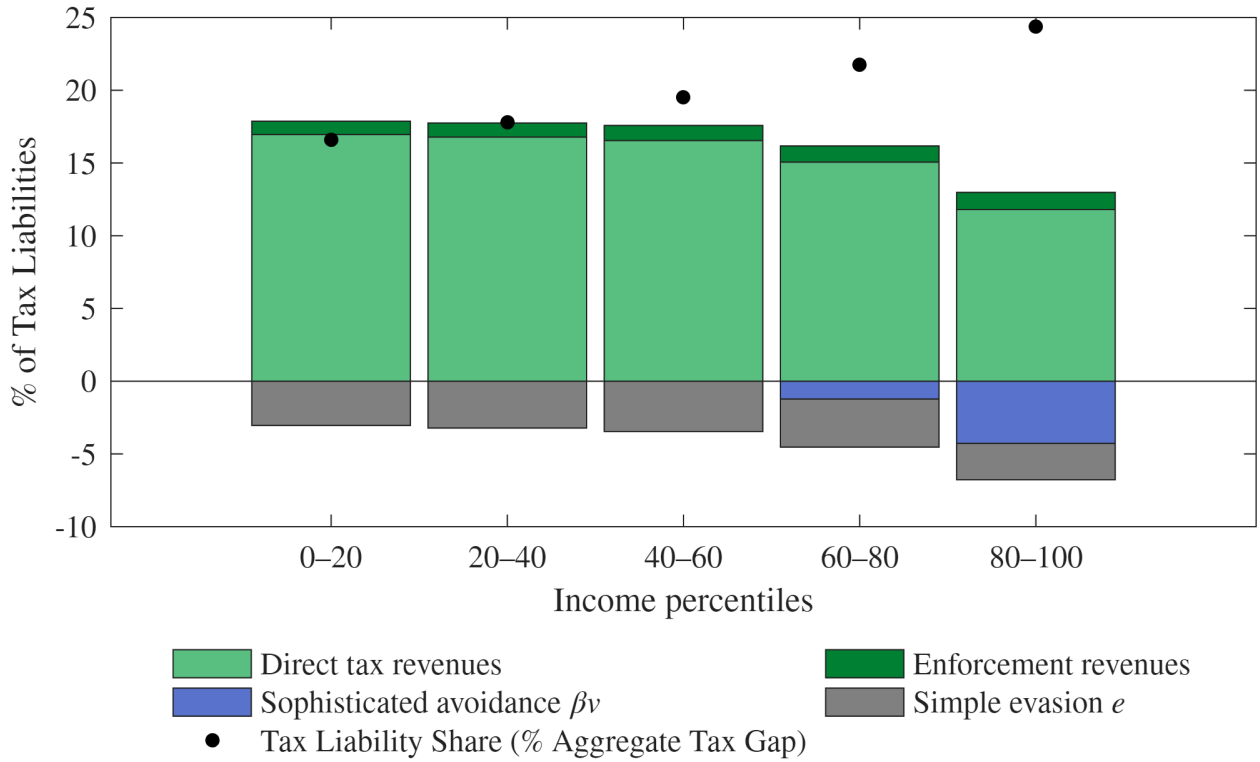
This figure plots tax revenues as a share of output for two stationary income distributions generated by different productivity volatilities,  $\sigma_a = 0.15$  (lower dispersion) and  $\sigma_a = 0.35$  (higher dispersion). Although the more dispersed economy exhibits a slightly higher revenue-maximizing tax rate, its peak revenue is substantially lower. Greater dispersion shifts a larger share of taxable income either toward low-income households with a small base or toward high-income households that rely on sophisticated evasion strategies, reducing the portion of the tax base that is effectively enforceable. As a result, higher inequality flattens the Laffer curve and lowers fiscal capacity, even when the location of the peak changes only modestly.

This contraction arises despite unchanged average productivity and reflects the compositional shift of the tax base toward agents with higher evasion propensities and lower enforcement elasticities.

**Evasion Concentration** Figure 7 documents how evasion becomes more concentrated at the top of the income distribution as inequality widens. The highest taxpayers increase

their total evasion as a share of all tax liabilities under high dispersion. Simple evasion  $e^*$  grows moderately across all groups, while sophisticated evasion  $v^*$  expands disproportionately among the richest decile due to the convexity of avoidance costs  $f(v) = \chi_0 v + \frac{\chi_1}{2} v^2$ . This reallocation amplifies the “gray-area” segment of the tax gap, further flattening the fiscal-capacity curve.

Figure 7: **Concentration of total evasion by income percentile. Higher dispersion reallocates the tax gap toward top-income agents, raising the top income groups share of total evasion.**



**Interpretation** The results show that inequality erodes fiscal capacity through two reinforcing channels: (i) *composition*—a larger share of revenue depends on agents with lower compliance elasticity; and (ii) *concentration*—a growing top-end share of sophisticated evasion that remains partly undetected. Together these mechanisms imply

$$\frac{\partial \text{FC}}{\partial \sigma_a} < 0, \quad \frac{\partial \text{Evasion Concentration}}{\partial \sigma_a} > 0,$$

consistent with the analytical results in Section 2. In other words, greater inequality endogenously constrains governments' revenue-raising capacity even in the absence of aggregate productivity losses, under the existence of an exploitable legal gray area:  $a \geq \frac{\lambda_0}{\tau\beta}$  for.

## 5 Discussion

The results in Section 4 underscore the critical role that the income distribution of taxpayers  $f(a, k)$  plays in shaping tax evasion behavior and its implications for government revenue collection over time. This section delves into the mechanisms driving these results, the policy implications, broader insights for tax enforcement, and the limitations of the model.

### 5.1 Main Mechanisms

#### 5.1.1 Unequal Capital Growth Dynamics.

The unequal capital growth dynamics observed across agents within the joint productivity and capital distribution  $f(a, k)$  arise from their higher propensity to optimally choose riskier strategies, such as increased evasion and greater investment in risky portfolio shares, to maximize their future income streams  $y_t$ . These dynamics are largely driven by agents' HARA utility preferences, where risk aversion decreases with income. Consequently, agents' optimal strategies to maximize consumption through future income streams  $y_t$  are highly dependent on their initial productivity  $a_0$  and capital  $k_0$ . Agents with higher  $a_0$  and  $k_0$  can afford to engage in both higher levels of total evasion and riskier portfolio investments. As a result, their total evasion strategy,  $\nu_t^* + e_t^*$ , increases monotonically with income  $y_t$ .

Increased perceptions of audit probability or fines lead to less significant reductions in total evasion for wealthier agents. Consequently, total evasion increases with income as wealthier taxpayers' optimal strategies become less sensitive to the expected costs of being audited. This further reinforces riskier behavior among wealthier agents. Increased evasion creates a feedback loop where disposable capital, after financing future consumption streams,

leads to riskier investments, higher returns, and further increases in future evasion.

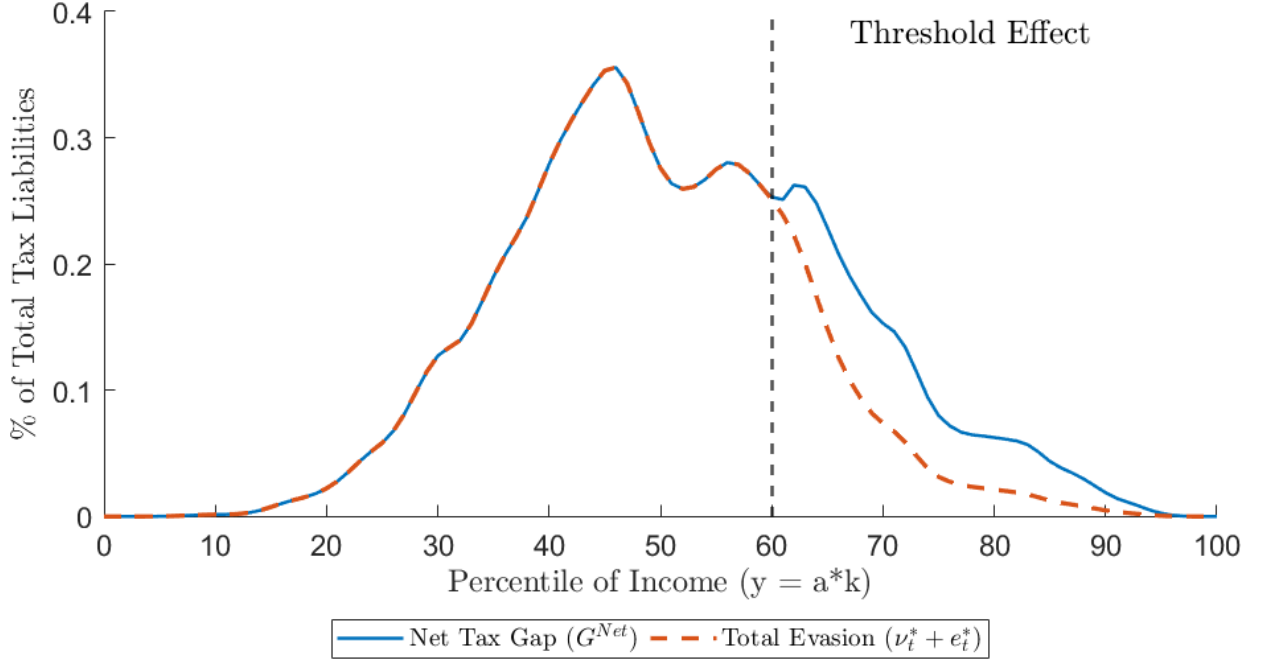
Nonetheless, the mean-reversion process driven by  $\mu$  ensures that total evasion does not grow indefinitely in the long run and eventually converges. This ensures that all optimal paths converge to a stationary joint stationary distribution for all agents. Thus, for a given set of optimal evasion paths, random audits will always remain effective at reducing evasion in the short term across all income groups.

### **5.1.2 Threshold Effects Caused by Fixed Costs Lead to Disproportionate Effects on the Tax Gap.**

Taxpayers with sufficient productivity ( $a_t$ ) and capital ( $k_t$ ) to surpass the threshold  $a_t \geq \frac{\chi_0}{\tau\beta}$  disproportionately contribute to the tax gap. This threshold stems from the fixed costs  $(\chi_0, \chi_1)$  required for sophisticated evasion strategies, creating a discontinuity in evasion behavior across income levels and further limiting the government's capacity to close the tax gap through increased random audits or fine rates.

As more capital is allocated to cover fixed costs  $(\chi_0, \chi_1)$ , taxpayers above the threshold significantly increase their total evasion. This results in disproportionately greater impacts on the net tax gap, as a larger share of the income generated by these agents is not collected by the government.

Figure 8: **Distribution of Net Tax Gap and Evasion Contributions** This figure depicts the amount of evasion and net tax gap contributed to the total amounts by income group, weighted by the amount of wealth held by agents across the distribution.



While total evasion may grow monotonically with income, contributions to the net tax gap do not. The effectiveness of collecting fines or reducing evasion decreases for wealthier agents. This phenomenon is illustrated in Figure 8, which shows the long-run distribution of agents' contributions to both total evasion and the net tax gap under baseline assumptions (Section 4.3).

The threshold effect introduces a non-monotonic decrease in the ability of governments to close the tax gap among wealthier taxpayers. Traditional deterrence policies, such as random audits ( $\lambda$ ) and fines ( $\eta$ ), have no impact on sophisticated evasion strategies in the short term, leading to increased evasion behaviors despite greater detection risks. This further diminishes their effectiveness in reducing total evasion among the wealthy.

The amount of taxable income subject to possible abuse, quantified as  $\beta\nu_t y_t \tau$ , reduces

the government’s capacity to reduce the total tax gap. The amount of income in an economy subject to this, the legal gray area depends on the distribution of  $a_t$  agents in an economy, the fixed costs to engage in sophisticated evasion  $\chi_0, \chi_1$ , the tax rate  $\tau$ , and more importantly, the government’s capacity to enforce fines  $\beta$ . Agents with average initial productivity  $\bar{a}_{Init}^{Average}$  contribute disproportionately to the net tax gap and follow income and evasion paths similar to those agents who started with high productivity. This happens as in the high enforcement setting, agents  $\bar{a}_{Init}^{Average}$  do not cross threshold  $a_t \geq \frac{\chi_0}{\tau\beta}$ , but do so in the low enforcement one. On the other hand, agents who start low levels of productivity  $a_{Init}^{Low}$ , follow very similar evasion, capital and income paths in both enforcement settings. More importantly, they do not contribute disproportionately to the tax gap as the other agents who cross this threshold do.

Thus, it can be seen that higher fine enforcement capacity ( $\beta$ ) reduces the legal gray area where taxpayers exploit ambiguous boundaries between evasion and avoidance, strengthening the government’s ability to collect revenues from wealthier taxpayers over time.

## 5.2 Policy and Broader Implications

The model’s mechanisms underscore critical insights into how enforcement policies influence tax compliance across different income groups and over time. The effectiveness of tax deterrence policies, such as random audits, fines, and fine enforcement capacity ( $\beta$ ), are intricately tied to the underlying distribution of agents ( $f(a, k)$ ) and their expected capital accumulation trajectories.

**Heterogeneous Effects of Enforcement Policies:** The aggregate size and magnitude of deterrence depend heavily on the initial distribution of agents and the uneven impact of risk determinants such as  $\mu, \sigma, \pi, \lambda$ . Wealthier agents, with greater capital and productivity levels, are uniquely positioned to engage in riskier strategies, accumulating more wealth over time. This dynamic reduces the overall efficacy of conventional policies like random audits

( $\lambda$ ) and fines ( $\eta$ ) as income inequality rises.

**Threshold effects from fixed costs** ( $a_t \geq \frac{\lambda_0}{\tau\beta}$ ) concentrate sophisticated evasion and disproportionate net tax gap losses among high-income taxpayers and make traditional enforcement less effective for this group. The more amount of total income in the economy is concentrated in the hands of tax payers who are above this threshold, the higher the legal gray area of an economy may be, reducing the efficacy of classic evasion deterrence tools to collect income over time and in the short-run.

**Balancing Short-Term and Long-Term Policy Trade-offs ( $\lambda$  and  $\beta$ ):** While random audits ( $\lambda$ ) remain effective in reducing simple evasion among lower-income taxpayers, their short-term impact diminishes when addressing wealthy individuals who rely on sophisticated evasion strategies. The effectiveness of enforcement depends on the government's ability to influence expectations about future income streams. Specifically, increasing  $\beta$  alters agents' perceptions that illegal income, particularly that exploiting legal gray areas, will face higher fines and reduced loopholes. This adjustment reduces the incentive to engage in sophisticated evasion. However, higher  $\beta$  may also dampen capital accumulation among high-income taxpayers in the long run. Additionally, the choice of enforcement strategy must consider the expected implementation costs, which are not explored within this paper. These costs, ranging from administrative expenses to the economic impact of altered capital accumulation, are pivotal in determining the overall feasibility and effectiveness of policy measures. Thus, the optimal policy lies in balancing these trade-offs—using  $\lambda$  and fines effectively in the short run, especially for taxpayers unlikely to cross the  $\beta$  threshold, while strengthening institutional and legal enforcement to achieve sustained compliance in the long run.

### 5.2.1 Policy Recommendations

The insights from the model suggest several key recommendations for addressing tax evasion and closing the tax gap more effectively.



**Increase random audits to deter simple evasion, but complement them with stronger fine enforcement for high-income individuals.** In the short term, increasing random audits ( $\lambda$ ) can effectively reduce simple evasion, particularly among lower-income taxpayers who are responsive to detection risks and lack the resources for sophisticated strategies, as demonstrated in Table 1. However, Figure 3 shows diminishing returns when applying random audits to high-income individuals who exploit sophisticated evasion strategies, both individually and in aggregate. This underscores the necessity of coupling random audits with policies that enhance the government’s fine enforcement capacity or demonstrate the ability to utilize new reporting mechanisms effectively. As corroborated in the past literature, increased detection rates that improve credible third-party information reporting, combined with increased random audits rates, can still be particularly effective; as low-income tax payers do not expect their income processes to lead them to a point where they can exploit the legal gray areas available past the threshold where sophisticated evasion becomes available (Kleven et al., 2011). As such, credible increases in the likelihood that detection will lead to a fine through random audits may be especially impactful in countries with low levels of third-party information reporting or where many agents cannot afford the fixed costs of sophisticated evasion.

**Enhance fine enforcement capacity ( $\beta$ ) to address sophisticated evasion and reduce future evasion opportunities.** Increasing fine enforcement capacity  $\beta$  is critical for reducing the legal gray area exploited by high-income individuals, as shown in Figures 8 and 5. Enhanced fine enforcement reduces opportunities for high-income taxpayers to avoid detection and increases the penalties for non-compliance. Achieving this requires simplifying tax codes and allocating resources to the tax collecting agencies and the judiciary for processing complex evasion cases more effectively Gamannossi degl’Innocenti et al. (2022). Additionally, addressing information asymmetries through targeted audits and third-party reporting requirements on the rich, improves the quality of fine enforcement revenues while

preventing future income from being diverted into sophisticated evasion. By reducing the size of the economy subject to legal gray areas, these measures also enhance the proportional deterrence effect of random audits and fines across all income groups.

**Integrate domestic and global enforcement mechanisms for a cohesive tax compliance framework.** Effective tax enforcement requires a comprehensive strategy that aligns global transparency efforts with strengthened domestic legal frameworks and auditing capacity. Global reforms that enhance reporting requirements should be designed to support domestic auditing processes, ensuring that the newly acquired information is effectively leveraged to improve fine enforcement on evaders. Without a credible domestic threat that this information will lead to tangible penalties, high-income taxpayers may perceive audits as low-risk and continue evading taxes, as the expected gains from evasion in the future outweigh the potential costs of being fined. To address this, reforms must prioritize increasing transparency, enhancing fine enforcement, and closing legal ambiguities. By reducing the gray areas that enable sophisticated evasion strategies, these measures ensure the government can sustain revenue collection and strengthen its ability to deter evasion across all income groups.

These policy recommendations may have direct implications when analyzing the effects of global initiatives like FATCA and the CRS. If rich tax payers do not believe that increased in third party information reporting can be effectively used to fine sophisticated evasion strategies, and a massive amount of tax liabilities in the economy are subject to legal grey areas, then the rich tax payers evasion behavior will not be proportionately curtailed by a threat of an increase in auditing.

### **5.2.2 Policy Discussion through the Model: FATCA, CRS, and U.S. Proposals**

**Global Initiatives: FATCA and CRS** were landmark initiatives designed to address cross-border tax evasion by requiring foreign financial institutions to report information on

non-resident account holders. While these frameworks successfully reduced offshore evasion, they left domestic enforcement gaps unaddressed - IRS estimates of the total tax gap and its composition, have remain relatively unchanged over the years (See Figure 11 in the Appendix). Third party-information acquired by the newly required reporting standards enacted by them were perhaps not as effective in reducing the tax gap as the threat of domestic fine enforcement at home was not leveraged. Specifically:

- **FATCA and CRS focus on information sharing**, improving governments' ability to detect cross-border evasion. However, they do not target domestic evasion strategies or effectively enhance fine enforcement ( $\beta$ ) for high-income individuals detected in new cross-border domestic flow data acquired through the reporting mechanisms enacted by them.
- **Threshold Effects and Legal Gray Areas**: Figure 8 show how sophisticated evasion strategies dominate once taxpayers surpass the fixed cost threshold. FATCA and CRS may be less effective in curtailing evasion, as richer tax payers' can keep relying on exploiting domestic legal ambiguities rather than hiding income offshore, or rely on the government's inability to enforce fines under these new reporting mechanisms. These findings corroborate recent government assessments that the Internal Revenue service is still not prepared to enforce compliance with FATCA Treasury Inspector General for Tax Administration (TIGTA) (2018), as the IRS could not corroborate the data provided to them on U.S. taxpayers.

### 5.2.3 Proposed Domestic Legislation

The Biden administration's proposed financial reporting reforms, as outlined in "The American Families Plan Tax Compliance Agenda" U.S. Department of the Treasury (2021), may address these shortcomings by:

- **Increasing Fine Enforcement Capacity ( $\beta$ ) through better Funding**: By im-

proving the IRS’s ability to enforce fines domestically through improved funding and the ability to leverage third-party information to successfully target their audit efforts, this proposed legislation reduces the legal gray area that sophisticated evasion exploits. Third-party information that is already available through enhanced reporting established by FATCA, could finally be used to pose a more credible threat to richer tax payers, making the threat of random audits increasingly effective in deterring their use of both simple and sophisticated evasion strategies.

- **Improving Domestic Third-Party Reporting:** The proposal mandates that domestic banks and financial institutions report aggregate inflows and outflows for individual accounts, rather than detailing income by type. Additionally, extending reporting requirements to encompass new financial assets, such as cryptocurrency transactions, would enable the IRS to monitor compliance among high-value accounts over time. Discrepancies between bank-reported aggregates and taxpayer filings could be utilized to enhance audit effectiveness and fine enforcement on individuals.
- **Focusing on Domestic High-Income Taxpayers:** Unlike FATCA and CRS, which target cross-border evasion, this proposal prioritizes high-income taxpayers at home. By implementing improved reporting on financial flows and establishing exceptions for accounts below a low de minimis gross flow threshold, the IRS can focus its audit efforts on significant misreporting. This strategy addresses information asymmetries that facilitate under-reporting by high-income taxpayers, who often have the means to restructure income sources through professional services and absorb associated fixed costs. Consequently, the focus remains on wealthier individuals, aligning with the model’s emphasis on the necessity for stronger fine enforcement.

These measures may be particularly impactful in light of the model’s findings, as they improve the quality of fine enforcement and address critical gaps in existing frameworks. Figure 8 underscore the importance of  $\beta$  in closing the tax gap, demonstrating how enhanced

fine enforcement capacity and reduced ambiguity in domestic tax reporting complement global transparency efforts. Additionally, the new legislation effectively targets wealthy individuals financial flows domestically that exceed the fixed cost threshold for sophisticated evasion, thereby addressing a significant shortfall left by FATCA and CRS.

### 5.3 Limitations

Despite its contributions, the model has several limitations that warrant discussion:

- **Restrictive Assumptions:** The model assumes linear tax rates ( $\tau$ ) and uniform audit probabilities ( $\lambda$ ). In reality, high-income taxpayers are often subject to higher tax rates and more frequent audits, which, in this framework, have opposing effects on evasion behavior. While incorporating these dynamics would improve realism, the core insights of the model are unlikely to change significantly.
- **Limited Corroborating Data:** The model relies heavily on U.S. data, particularly estimates from Guyton et al. (2023). Although similar evasion patterns have been documented by Alstadsæter et al. (2019); Leenders et al. (2023); Keen and Slemrod (2017); Londoño-Vélez and Ávila Mahecha (2021) in Scandinavia, Norway, and Colombia, more comparable cross-country evidence is needed to generalize the findings.
- **Partial Equilibrium Setting:** The absence of supply-side modeling, such as the role of firms and financial intermediaries in facilitating evasion, limits the model’s ability to conduct a full welfare analysis. Incorporating these elements could provide a more comprehensive understanding of evasion dynamics.
- **Government Inaction:** The model does not account for the costs of auditing or increasing  $\beta$ , nor does it consider redistribution benefits. While these omissions limit the analysis of optimal government policies, the framework can be extended to include such considerations.

## 6 Conclusion

The old adage that "the poor evade and the rich avoid" remains partially true, but the paper's findings suggest a more nuanced view: "the poor and the rich evade, but the rich also avoid when they can"—placing the responsibility on governments to define and enforce the boundaries between legal avoidance and illegal evasion. This framework explains why tax deterrence policies, such as increasing random audits, may have limited or short-lived effects, particularly on high-income taxpayers who can exploit sophisticated evasion strategies. The model also highlights the critical role of fine enforcement capacity in sustaining compliance, especially among the wealthy. The U.S. calibration exercise validates the model's capacity to replicate observed patterns in the tax gap and its decomposition, providing a solid foundation to study the impacts of short- and long-term tax deterrence policies.

The results emphasize the importance of income heterogeneity and time dynamics in shaping the effectiveness of tax enforcement. While random audits can effectively deter simple evasion in the short run, their long-term impact diminishes as wealthy agents keep exploiting legal gray areas as long as they can afford it. Strengthening fine enforcement capacity emerges as a key lever for reducing the tax gap in the long term. Future research should explore the model in a general equilibrium setting, incorporating optimal government decisions that account for enforcement costs and redistribution effects. Additionally, examining the role of progressive taxation and auditing would provide further insights into designing equitable and efficient tax systems. This framework lays the groundwork for understanding how policy interventions can address persistent tax gaps and promote sustainable revenue generation.

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# Figures

Figure 9: **Comparison of Model Simulations and Estimates on US Unreported Income, by Percentile of Income (as % of True Income  $y_t$ ), Average 2006-2013.** The top panel shows the model estimates of unreported income, while the bottom panel presents the corresponding real data as estimated by Guyton et al. (2023). Both plots display the distribution of simple and sophisticated evasion across percentiles of income.

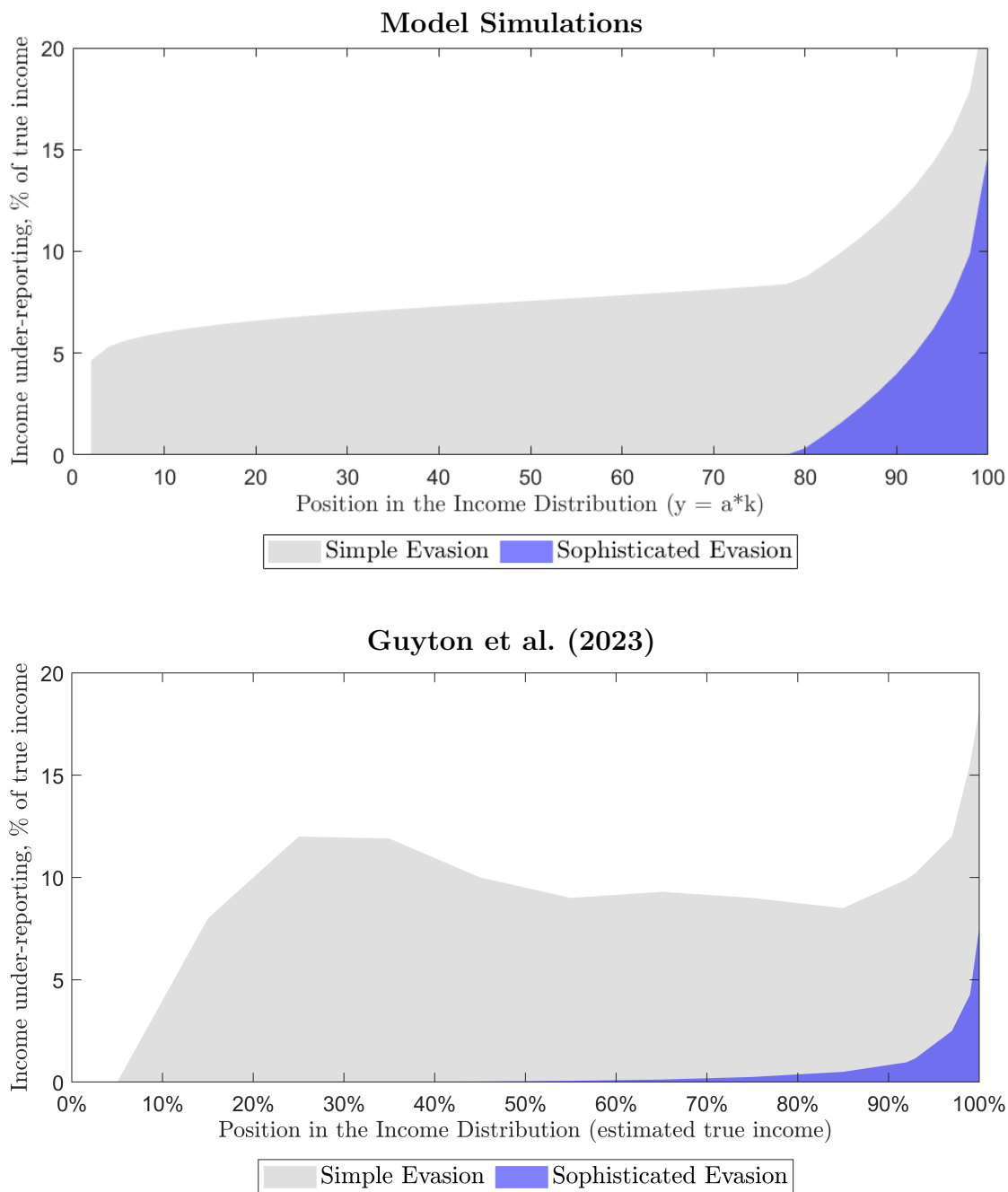


Figure 10: U.S. Tax Gap Decomposition Estimates over Time

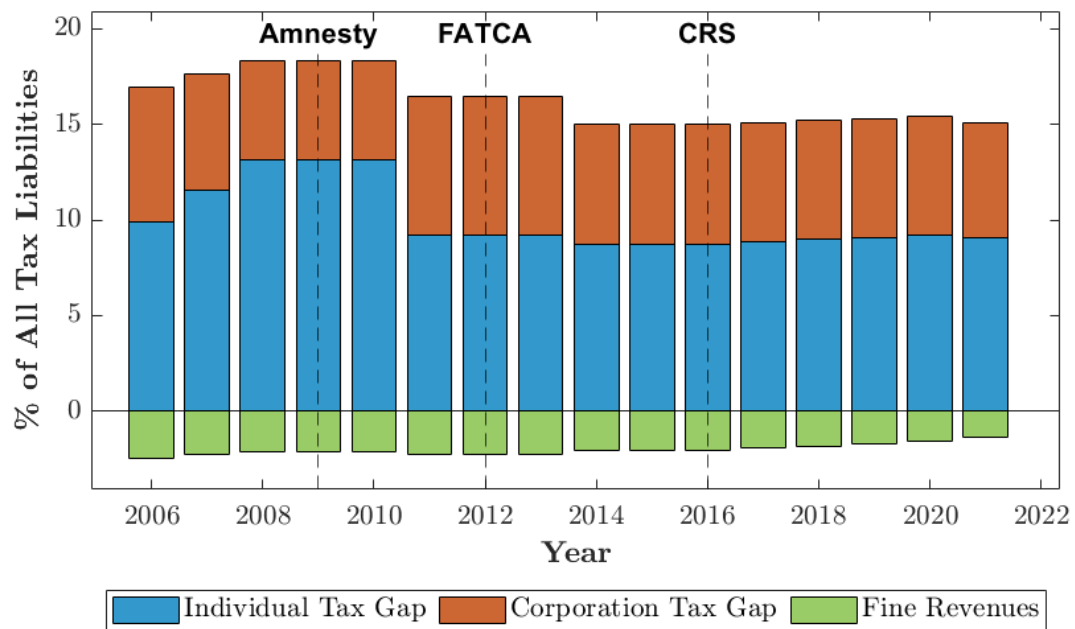


Figure 11: **Source:** IRS (2019) Estimates.

## Appendix A. Analytical Solution

Omitting time subscripts, the taxpayer's constrained optimization problem stated in eq.(8) satisfies the following Hamilton–Jacobi–Bellman equation (HJBE)<sup>12</sup>:

Entrepreneurs value function defined as  $V(k, a) := \max_{c, e, v} J(k, a; c, e, v)$  satisfies the following HJBE:

$$(\rho + \lambda + \phi(1 - \chi))V(k, a) = \max_{c, \theta, e, v} \left\{ \frac{(c - c_m)^{1-\gamma}}{1 - \gamma} + \lambda V(k(1 - a\eta\tau(e + (1 - \beta)v)), a) \right. \\ \left. + V_k k \left( a(1 - \tau(1 - e - v)) - f(v) - \frac{c}{k} + \pi\theta \right) + \frac{1}{2} V_{kk} k^2 \theta^2 \sigma^2 \right. \\ \left. \underbrace{V_a \kappa(a) + \frac{1}{2} V_{aa} \sigma_a^2}_{f(a; \sigma_a)} \right\}. \quad (26)$$

$$\left. \underbrace{V_a \kappa(a) + \frac{1}{2} V_{aa} \sigma_a^2}_{f(a; \sigma_a)} \right\}. \quad (27)$$

For our analytical solution let's we shall assume  $f(a; \sigma_a)$  fixed for all agents across their lives.

### Guess and verify solution

Suppose

$$V(k, a) = \frac{F(a)}{1 - \gamma} (k - H(a))^{1-\gamma}, \quad (28)$$

with  $F(a) > 0$ ,  $H(a) \geq 0$ . Then

$$V_k = F(a)(k - H(a))^{-\gamma}, \quad V_{kk} = -\gamma F(a)(k - H(a))^{-\gamma-1}.$$

The optimal controls are

$$c^*(k, a) = c_m + (k - H(a))F(a)^{-\frac{1}{\gamma}}, \quad (29)$$

$$\theta^*(k, a) = \frac{k - H(a)}{k} \frac{\pi}{\gamma \sigma^2}, \quad (30)$$

$$e^*(k, a) = \frac{k - H(a)}{a\eta\tau k} [1 - (\eta\lambda)^{1/\gamma}] - (1 - \beta)v^*(a). \quad (31)$$

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<sup>12</sup>See Karatzas and Shreve (1998).

The level function  $H(a)$  is pinned down by

$$H(a) = \frac{c_m}{a(1 - \tau + \tau\beta v^*(a)) - f(v^*(a))}. \quad (32)$$

Finally,  $F(a)$  satisfies

$$\begin{aligned} F(a)^{-1/\gamma} &= \frac{1}{\gamma} \left[ \rho + \lambda + \phi(1 - \chi) - \lambda(\eta\lambda)^{\frac{1-\gamma}{\gamma}} \right] \\ &\quad + \frac{\gamma - 1}{\gamma} \left\{ a(1 - \tau + \tau\beta v^*(a)) - f(v^*(a)) + \frac{1}{\eta} [1 - (\eta\lambda)^{1/\gamma}] + \frac{\pi^2}{2\gamma\sigma^2} \right\}. \end{aligned} \quad (33)$$

## Excess-capital dynamics

Defining  $z_t = k_t - H(a)$ , we obtain

$$\frac{dz_t}{z_t} = \mu(a) dt + \Sigma dZ_t + \Gamma d\Pi_t, \quad (34)$$

with

$$\begin{aligned} \mu(a) &= \frac{1}{\gamma} \left( a(1 - \tau + \tau\beta v^*) - f(v^*) \right) + \frac{1}{\gamma\eta} [1 - (\eta\lambda)^{1/\gamma}] \\ &\quad + \frac{\pi^2}{\gamma\sigma^2} \left( 1 + \frac{\gamma - 1}{2\gamma} \right) - \frac{1}{\gamma} \left( \rho + \lambda + \phi(1 - \chi) - \lambda(\eta\lambda)^{\frac{1-\gamma}{\gamma}} \right), \end{aligned} \quad (35)$$

$$\Sigma = \frac{\pi}{\gamma\sigma}, \quad (36)$$

$$\Gamma = (\eta\lambda)^{1/\gamma} - 1. \quad (37)$$

## Stationary distribution of excess capital

Let  $z_t = k_t - H(a)$ . From (??),  $z_t$  follows a geometric jump–diffusion. Denote the stationary pdf by  $p(z, a)$ . For  $z \neq z^* := k^* - H(a)$ , the Fokker–Planck equation is

$$\begin{aligned} 0 &= -\frac{d}{dz} (\mu(a)zp(z, a)) + \frac{1}{2} \frac{d^2}{dz^2} (\Sigma^2 z^2 p(z, a)) \\ &\quad + \lambda \left( \frac{1}{1 + \Gamma} p\left(\frac{z}{1 + \Gamma}, a\right) - p(z, a) \right) - \phi p(z, a), \end{aligned} \quad (38)$$

We guess a power-law form

$$p(z, a) = Cz^{-\alpha(a)}, \quad (39)$$

valid piecewise for  $z < z^*$  and  $z > z^*$ . Imposing mass preservation and continuity at  $z^*$ ,

$$\int_0^\infty p(z, a) dz = 1, \quad (40)$$

$$\lim_{z \rightarrow z^*-} p(z, a) = \lim_{z \rightarrow z^*+} p(z, a), \quad (41)$$

implies

$$p(z, a) = \begin{cases} C_1(a)z^{-\alpha_-(a)}, & z < z^*, \\ C_2(a)z^{-\alpha_+(a)}, & z > z^*, \end{cases} \quad (42)$$

where  $\alpha_-(a), \alpha_+(a)$  are roots of the characteristic equation

$$0 = \left( \frac{1}{2}\Sigma^2(2 - \alpha) - \mu(a) \right)(1 - \alpha) + \lambda \left( (1 + \Gamma)^{\alpha-1} - 1 \right) - \phi. \quad (43)$$

The coefficients  $C_1(a), C_2(a)$  are pinned down by 42 and 43, yielding

$$C_1(a)z_*^{-\alpha_-(a)} = C_2(a)z_*^{-\alpha_+(a)}, \quad (44)$$

$$\frac{C_1(a)}{1 - \alpha_-(a)} \left( z_*^{1-\alpha_-(a)} - 1 \right) + \frac{C_2(a)}{\alpha_+(a) - 1} z_*^{1-\alpha_+(a)} = 1. \quad (45)$$

## Appendix B. Numerical Solution

Omitting time subscripts, the taxpayer's constrained optimization problem stated in eq.(8) satisfies the following Hamilton–Jacobi–Bellman equation (HJBE)<sup>13</sup>:

$$\begin{aligned}
0 = & \frac{\partial V}{\partial t} - (r + \lambda) V + \frac{\partial V}{\partial k} a k (1 - \tau) + \frac{\partial V}{\partial a} \mu_a + \frac{1}{2} \frac{\partial^2 V}{\partial a^2} \sigma_a^2 + \\
& + \sup_{e,v} \left\{ \frac{\partial V}{\partial k} k [\tau a (e + v) - \phi v - f(v)] + \lambda V (k - \tau a k \eta (e + (1 - \beta)v)) \right\} \\
& + \sup_{\theta} \left\{ \frac{\partial V}{\partial k} k \theta \pi + \frac{1}{2} \frac{\partial^2 V}{\partial k^2} \theta^2 k^2 \sigma_a^2 + \theta k \frac{\partial^2 V}{\partial a \partial k} \sigma_a^2 \right\} + \sup_c \left\{ \frac{(c - c_m)^{1-\gamma}}{1 - \gamma} - \frac{\partial V}{\partial k} c \right\}
\end{aligned}$$

with complementary slackness condition  $\phi v = 0$ , where  $\phi$  is the Lagrangian Multiplier. Subsequently, its FOCs are:

$$\begin{aligned}
c : c &= c_m + \left( \frac{\partial V}{\partial k} \right)^{-\frac{1}{\gamma}} \\
e : \frac{\partial V}{\partial k} &= \eta \lambda \frac{\partial V (k - \tau a k \eta (e + (1 - \beta)v))}{\partial (k - \tau a k \eta (e + (1 - \beta)v))} \\
v : \frac{\partial V}{\partial k} \left( \tau a - \phi - \frac{\partial f(v)}{\partial v} \right) &= \lambda \frac{\partial V (k - \tau a k \eta (e + (1 - \beta)v))}{\partial (k - \tau a k \eta (e + (1 - \beta)v))} \tau A \eta (1 - \beta) \\
\theta : \frac{\partial V}{\partial k} k \pi + \frac{\partial^2 V}{\partial k^2} \theta k^2 \sigma_a^2 + k \frac{\partial^2 V}{\partial a \partial k} \sigma_a^2 &= 0
\end{aligned}$$

Substituting the second FOC in the third and rearranging yields:

$$\begin{aligned}
e : \frac{\partial V}{\partial k} &= \eta \lambda \frac{\partial V (k - \tau a k \eta (e + (1 - \beta)v))}{\partial (k - \tau a k \eta (e + (1 - \beta)v))} \\
\frac{\frac{\partial V}{\partial k} \left( \tau a - \phi - \frac{\partial f(v)}{\partial v} \right)}{\tau a (1 - \beta)} &= \lambda \frac{\partial V (k - \tau a k \eta (e + (1 - \beta)v))}{\partial (k - \tau a k \eta (e + (1 - \beta)v))} \eta
\end{aligned}$$

**Solving the model:** Next, we guess (and verify) that the value function takes the following form:

$$V = F(t, a)^b \frac{(k - H(t, a))^{1-\gamma}}{1 - \gamma}$$

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<sup>13</sup>See Karatzas and Shreve (1998).

where  $b$  is a free coefficient to be determined at convenience. Omitting functional dependence, the remaining FOCs entail the following optimal policies:

$$c^* = c_m + F^{-\frac{b}{\gamma}} (k - H)$$

$$e^* = \frac{(k - H)}{\tau a \eta k} \left( 1 - (\eta \lambda)^{\frac{1}{\gamma}} \right) - (1 - \beta) v^*$$

$$\theta^* k = (k - H) \left( \frac{\pi}{\gamma \sigma_a^2} + b \frac{\partial F}{\partial a} \frac{1}{F} \right) + \frac{\partial H}{\partial a}$$

Note that tax avoidance and evasion are substitutes (see  $e^*$ ). Evasion is increasing in net worth ( $k$ ) but decreasing in productivity  $a$  (directly); unclear indirectly (through  $H(a)$ ). Substituting the guesses, the optimal policies in the HJBE, setting  $b = 1$  and rearranging yields the following PDE:

$$\begin{aligned} (r + \lambda) F \frac{(k - H)^{1-\gamma}}{1 - \gamma} &= \frac{\partial F}{\partial t} \frac{(k - H)^{1-\gamma}}{1 - \gamma} - F(k - H)^{-\gamma} \frac{\partial H}{\partial t} + F(k - H)^{1-\gamma} A(1 - \tau) + \\ &+ F(k - H)^{-\gamma} H A(1 - \tau) + \frac{\partial F}{\partial a} \frac{(k - H)^{1-\gamma}}{1 - \gamma} \mu_a - F(k - H)^{-\gamma} \frac{\partial H}{\partial a} \mu_a + \\ &+ F(k - H)^{1-\gamma} (\tau A \beta v^* - \phi v^* - f(v^*)) + \lambda F(k - H)^{1-\gamma} \frac{(\eta \lambda)^{\frac{1-\gamma}{\gamma}}}{1 - \gamma} + \\ &+ F(k - H)^{-\gamma} H (\tau A \beta v^* - \phi v^* - f(v^*)) + F(k - H)^{1-\gamma} \left( 1 - (\eta \lambda)^{\frac{1}{\gamma}} \right) + \\ &+ \frac{1}{2} b \frac{\partial^2 F}{\partial a^2} \frac{(k - H)^{1-\gamma}}{1 - \gamma} \sigma_a^2 - \frac{1}{2} F(k - H)^{-\gamma} \frac{\partial^2 H}{\partial a^2} \sigma_a^2 + \\ &+ F(k - H)^{1-\gamma} \frac{\pi^2}{\gamma \sigma_a^2} + F^b(k - H)^{-\gamma} \pi \frac{\partial H}{\partial a} + \\ &+ \frac{\pi}{\gamma \sigma_a^2} \frac{\partial F}{\partial a} (k - H)^{1-\gamma} \sigma_a^2 + \frac{\pi}{\gamma \sigma_a^2} \gamma F^b(k - H)^{-\gamma} \frac{\partial H}{\partial a} \sigma_a^2 + \\ &+ F^{1-\frac{1}{\gamma}} (k - H)^{1-\gamma} \frac{\gamma}{1 - \gamma} - (F(k - H)^{-\gamma} c_m) + \end{aligned}$$



$$-\frac{\gamma}{2}F(k-H)^{1-\gamma}\left(\frac{\pi}{\gamma\sigma_a^2}\right)^2\sigma_a^2-\frac{\gamma}{2}F(k-H)^{-\gamma}\frac{\pi}{\gamma\sigma_a^2}\frac{\partial H}{\partial a}\sigma_a^2$$

Similar to Menoncin and Vergalli (2021), separating the PDE into two equations containing  $(k-H)^{1-\gamma}$  and  $(k-H)^{-\gamma}$  yields the following two PDEs:

$$\rho_F F = F^{1-\frac{1}{\gamma}}\gamma + \frac{\partial F}{\partial t} + \left(\mu_a + \frac{1-\gamma}{\gamma}\pi\right)\frac{\partial F}{\partial a} + \frac{1}{2}\frac{\partial^2 F}{\partial a^2}\sigma_a^2$$

where

$$\rho_F := (1-\gamma)\left(\frac{r+\lambda}{1-\gamma} - a(1-\tau) - (\tau a\beta v^* - \phi v^* - f(v^*)) - (1-\gamma)\lambda(\eta\lambda)^{\frac{1-\gamma}{\gamma}} + \left(1 - (\eta\lambda)^{\frac{1}{\gamma}}\right) + \frac{1}{2\gamma}\frac{\pi^2}{\sigma_a^2}\right)$$

and

$$H\rho_H = c_m + \frac{\partial H}{\partial t} + \frac{\partial H}{\partial a}(\mu_a - \pi) + \frac{1}{2}\frac{\partial^2 H}{\partial a^2}\sigma_a^2$$

where  $\rho_H := A(1-\tau) + \tau A\beta v^* - \phi v^* - f(v^*)$ , with boundary conditions  $\lim_{t \rightarrow \infty} H_t < \infty$  and  $\lim_{t \rightarrow \infty} F_t < \infty$ .

The solutions of these two PDEs have the following Feynman-Kac equations:<sup>14</sup>

$$H_t = \mathbb{E}_t^{\mathbb{Q}} \left[ \int_t^\infty c_m e^{\int_t^s \rho_{H,u} du} ds \right]$$

with

$$dZ_a^{\mathbb{Q}} = \frac{\pi}{\sigma_a} dt + dZ_a$$

and

$$F_t = \mathbb{E}_t^{\gamma} [G_t]$$

where  $G_t$  satisfies the following ODE:

$$\frac{dG_t}{dt} = \rho_{F,t}G_t - G_t^{1-\frac{1}{\gamma}}\gamma$$

with boundary condition  $\lim_{t \rightarrow \infty} G_t = 0$ . This last equation is a Bernoulli equation, which can be linearized using the following transform:

$$U = G^{\frac{1}{\gamma}}$$

which implies

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<sup>14</sup>See Yong and Zhou (1999); Øksendal (2000).

$$\frac{dU_t}{dt} = \frac{1}{\gamma} U_t \rho_{F,t} - 1$$

Integrating yields

$$U_t = \int_t^\infty e^{-\frac{1}{\gamma} \int_t^s \rho_{F,u} ds} dt$$

and thus

$$F_t = \mathbb{E}_t^{\mathbb{Q}_\gamma} \left[ \left( \int_t^\infty e^{-\frac{1}{\gamma} \int_t^s \rho_{F,u} ds} dt \right)^\gamma \right]$$

where

$$dZ_a^{\mathbb{Q}_\gamma} = \frac{\gamma - 1}{\gamma} \frac{\pi}{\sigma_a} dt + dZ_a.$$

**Numerical Solution:** We compute  $F(a, t)$  and  $H(a, t)$  and thus the joint distribution of  $(k_t, a_t)$  and the optimal policies  $\{e_t^*(k_t, a_t), v_t^*(k_t, a_t), \theta_t^*(k_t, a_t), c_t^*(k_t, a_t)\}$  by numerical (Monte Carlo) simulation. To do this, we follow the following steps:

1. **Step 1:** Solve  $F$  and  $H$  as functions of  $a$  over a suitable grid.
2. **Step 2:** Simulate  $(k, a)$  using the optimal policy functions over a long time horizon. Discard the first  $N$  simulations and use the ergodic theorem to interpret time averages as cross-sectional net-worth productivity distributions (space averages).

**Deriving Evasion and Income:** Taxpayer's total evasion behavior,  $\bar{E} = \nu_t + e_t$ , increases with their net worth  $k_t$  and income  $y_t = a_t k_t$ . Total evasion increases with total income as:

$$\frac{\partial(\bar{E})}{\partial y} = \frac{(1 - \eta\lambda) \left( 1 - \left( \frac{\eta\lambda}{\tau} \right)^{1/\gamma} \right)}{\eta y} \left[ \underbrace{\frac{H(a, t)}{y}}_{\text{A) Simple Evasion Gains}} + \underbrace{\frac{H(a, t) \left( \frac{\beta\tau(y/k) - \chi_0}{k\chi_1} + \frac{1-\tau}{k} \right)}{\left( (y/k)(1-\tau) + \frac{(\tau(y/k)\beta - \phi - \chi_0)^2}{2\chi_1} \right)}}_{\text{B) Sophisticated Evasion Gains}} \right] + \underbrace{\frac{\beta^2\tau}{k\chi_1}}_{\text{C) Additional Sophisticated Evasion Growth}} \quad (46)$$

Where  $\frac{\partial(\bar{E})}{\partial y} > 0$ , even when we assume the minimum consumption level  $c_m = 0$ <sup>15</sup>. However, the size and magnitude of the policy parameter's effect on agent total evasion behaviour are unclear.

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<sup>15</sup>As  $H(a, t) \approx \frac{c_m}{a_t(1-\tau) + \frac{(\tau a \beta - \phi - \theta_0)^2}{2\theta_1}}$